Discontinuity Stress Analysis Of Metallic Pressure Vessel Using Finite Element Method

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Abstract

This project investigated the stresses developed in a thick-walled cylinder for rocket motor case under internal pressure. Stress analysis used the finite element method with ANSYS software for rocket motor case selection. This study focus on structural elastic analysis of thick-walled pressure vessels since it is a common design practice to aim at maintaining the induced stresses within the elastic region. However, pressure vessels operate under complex environments such as high pressure which may lead to gross plastic deformation and subsequent failure. In process, the pressure vessel is pressurized beyond the yield point. As a result, the conventional elastic analysis will not be applicable at internal pressures above the yield point. Therefore, it is important to examine the structural integrity of a thick-walled pressure vessel in both elastic and plastic state of the material.

In this study, FE static structural analysis of a presumably untracked thick-walled solid rocket motor case has been presented, where stress distribution within the motor case wall and the resulting material deformation were investigated using ANSYS 19.2. Motor case has been designed with uniform model of the same internal and external diameter, and motor case with diameter change at both sides is modeled to investigate the effect of the diameter change or shape discontinuity on the resulting of stresses and deformation using ANSYS program by applying internal pressure varying from 50 Bars to 350 Bar. Von Mises yield criteria were used by ANSYS program and calculated Von Mises stresses were compared; the results are close for elastic analysis. The results show that the Von Mises stresses was high for discontinues shape of motor case compared by the uniform motor case (constant thickness).

(Key words)

Pressure Vessel , Finite Element Method:FEM, ANSYS Software, Thick-Walled Pressure Vessel

1. Introduction

Pressure vessels are commonly used in daily life for different purposes ranging from domestic use for storage of hot water and natural gas to industrial applications for the storage of various high pressure gases and liquids. Pressure vessels should be carefully designed to avoid failure which may cause loss of life and money. Depending upon the application, the design of pressure vessels may complex due to complex geometry, combined structural and thermal loads and aggressive environment. In such situations, analytical solutions will become complex and the designer often use approximate solutions for design. Various standards are also available for the design of pressure vessels such as ASME code VIII and API codes; however, they result in conservative designs. Also for non-standard shapes and intersections and geometrical discontinuity, limit load and stress concentration formulae are not available[1]. Thick-walled pressure vessels are used for a variety of applications where large pressures are to be withstood. Pressure vessels are leak-proof containers. They may be of any shape and range from beveragebottles to the sophisticated ones encountered in engineering construction. Discontinuity always appears with changing section in any mechanical component like pressure vessel. Finite Element Analysis (FEA) provides approximate solutions for complex problems and widely used for structural analysis. It involves discretization of the structure, application of loads, solution and post processing. Various commercial software's like ANSYS, NASTRAN, and ABAQUS are available for FEA. Pressure vessels are usually subjected to high pressures and temperatures, which may be of constant time duration or frequent cycles[1]. Such vessels have a multi-axial stress situation where failure is not governed by the individual components of stress but by some combination of all the stress components. Therefore, failure would occur when the stress state somewhere in the wall material exceeds some failure criterion[2]. It is therefore important to be able to understand and quantify or resolve the stresses in a pressure vessel. The most commonly used factor in the design of pressure vessels is that of maintaining the induced stresses within the elastic region of the construction material in order to avoid excessive plastic deformation or rupture when the yield point is exceeded.[3].Consequently, this has made the study of stresses in pressure vessels to be mainly focused on elastic analysis. Therefore, the conventional elastic analysis of thick-walled pressure vessels would be applicable for internal pressures up to yield point, while in practice, such vessels often undergo pressures above the yield strength of the material[2]. In addition, pressure vessels operate under complex environments such as high pressure and temperature, which may lead to gross plastic deformation and subsequent failure[4]. Therefore, it is important to design and examine the structural integrity of a thick-walled pressure vessel in both elastic and plastic state of the material. This ensures that the full use of the load carrying capacity of the material is accounted for in assessing the structural integrity of the pressure vessel [1].Majority of the studies in the analysis of pressure vessels finds their origins in Lethnitskii's approach.[5] H. Al-Gahtani et al.[6]

Fig..1 Pressure Vessel

Thick walled pressure vessels are those which have the thickness to internal radius ratio more than in the current industrial scenario. The radial shear stress is considered. The hoop stress varies parabolic ally over the wall thickness. Examples are gun barrels, high-pressure vessels in oil-refining industry. Analytical treatment is complex and accurate. A thick cylinder is statically indeterminate. State of stress is biaxial.[7] Thin walled pressure vessels are those which have the thickness to internal radius ratio less than .The hoop stress is assumed to be uniformly distributed over the thickness. Examples are types gas storage tanks. Analytical treatment for stresses is simple and approximate. A thin cylinder is statically determinate. State of stress is membrane i.e. biaxial.[7]

1.2 Discontinuity Stresses

Vessel sections of different thickness, material, diameter, and change in directions would all have different displacements if allowed to expand freely. However, since they are connected in a continuous structure, they must deflect and rotate together. The stresses in the respective parts at or near the juncture are called discontinuity stresses. Discontinuity stresses are necessary to satisfy compatibility of deformation in the region. They are local in extent but can be of very high magnitude. Discontinuity stresses are ‘‘secondary stresses’’ and are self-limiting. That is, once the structure has yielded, the stresses are reduced. In average application they will not lead to failure. Discontinuity stresses do become an important factor in fatigue design where cyclic loading is a consideration.

1.3 Solid Rocket Motor Case (SRMC)

In a solid rocket motor (SRMC), the propellant consists of one or more pieces mounted directly in the motor case, which serves both as a propellant tank and as combustion chamber. The propellant is usually arranged to protect the motor case from heating. Most modern propellant charges are formed by pouring viscous mix into the motor case with suitable mould fixtures. The propellant solidifies and the mould fixtures are removed, leaving the propellant bonded to the motor case with suitably shaped perforation down the middle. During operation, the solid burns on the exposed inner surfaces.

These burns away at a predictable rate to give the desired thrust. solids are often used as additional strap- on boosters to increase payload capacity or as spin-stabilized add-on upper stages when higher-than-normal velocities are required. [8]

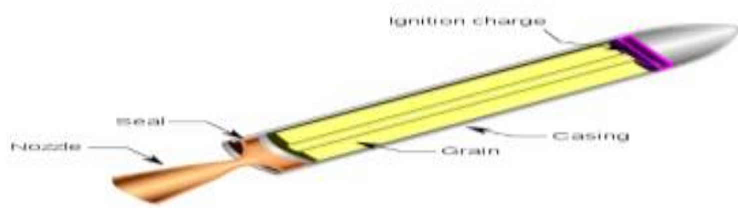

Fig..2 Solid Motor Components

A simple solid rocket motor consists of a casing, nozzle, grain (propellant charge), and igniter. The grain behaves like a solid mass, burning in a predictable fashion and producing exhaust gases.

2. Material And Method

5.1 Geometrical Dimensions Of Case Study

- Internal diameter of model is 235 mm and an outer diameter of 267 mm were taken for a model. Length of the model was taken as 2114 mm surface were taken and Figure (3) shows the geometry of the model with different diameters at two ends (discontinues shape). Symmetry boundary conditions were invoked such that only a model of the pressure was used with appropriate boundary conditions applied.

Since hoop stress (first principal stress) is always the maximum among the three resulting principal stresses, its distribution in the pressure vessel wall was examined.

- Von Mises failure criterion is said to be relatively more accurate in predicting failure in pressure vessels and therefore the resulting Von Mises stress was also investigated in this study.

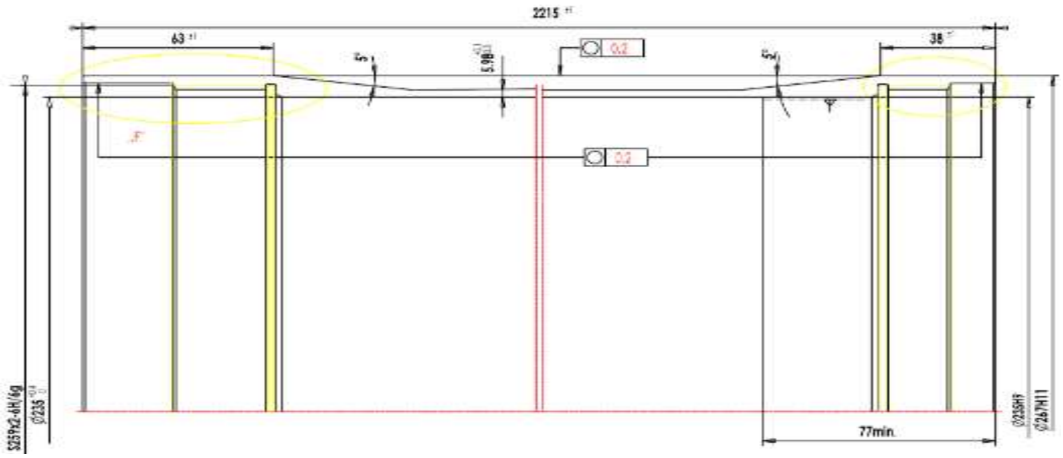

Fig. .3 Geometry Of The Model Case Study

5.2 Theoretical Calculation

Cylindrical or spherical pressure vessels (e.g. hydraulic cylinders, gun barrels, pipes, boilers and tanks) are commonly used in industry to carry both liquids and gases under pressure. When the pressure vessels are exposed to this pressure, the material comprising the vessel is subjected to pressure loading, and hence stresses, from all directions. The normal stresses resulting from this pressure are functions of the radius of the element under consideration, the shape of the pressure vessel (i.e., open ended cylinders, closed end cylinders, or sphere) as well as the applied pressure. A cylindrical pressure with wall thickness (t) and inner radius is considered Figure (4). A gauge pressure, exists within the vessel by the working fluid (gas or liquid).

For an element sufficiently removed from the ends of the cylinder and oriented as show in Figure (4), two types of normal stress are generated: Tangential stress or hoop stress (σ_1) , and longitudinal stress or axial stress (σ_2), that both exhibit tension of the material. Tangential stress or hoop stress (σ_1): The average stress acting on a cross section area of the vessel. For the hoop stress, consider the pressure vessel section by planes sectioned by plan C in Figure (5). A free body diagram of a half segment along with the pressurized working fluid.

Note that only the loading in the x-direction is show and that the internal reactions in the material are due to hoop stress acting on incremental areas section A, produced by the pressure acting on the project area, section A.

- **Elastic State Of Thick-Walled Pressure Vessel:**
- Hoop (tangential) stress (σ_1)
- Longitudinal (axial) stress (σ_2)
- Radial stress (σ_3)

The following derived equations provide the relationship between the internal pressure (p_i) and the various principal stresses:

$$(\sigma_1) = \frac{p_i^2 * r_i^2}{r_o^2 - r_i^2}\left(1+\frac{r_o^2}{r^2}\right) \quad (1)$$

$$(\sigma_3) = \frac{p_i^2 * r_i^2}{r_o^2 - r_i^2}\left(1-\frac{r_o^2}{r^2}\right) \quad (2)$$

$$(\sigma_2) = \frac{p_i * r_i^2}{r_o^2 - r_i^2} \quad (3)$$

Fig..4 Cylindrical Pressure Vessel

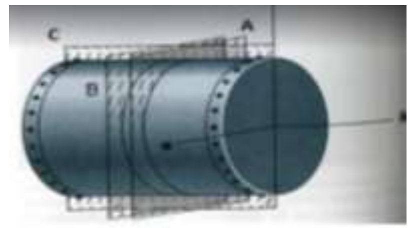

Fig..5 Cylindrical Pressure Vessel Showing Coordinate Axis And Cutting Planes (A, B and C)

➢ Elastic-Plastic State Of Thick-Walled Pressure Vessel

In thick-walled pressure vessels, stress is directly proportional to strain up to yield point. Beyond the yield point, there comes a phase (elastic- plastic) in which partly the material is elastic and partly plastic [1]. As the internal pressure is increased beyond the yield point, the plastic region also increases proportionally to the pressure until the whole material becomes plastic. The stress state of the plastic region with (n) Strain hardening exponent show as number 2 in Figure (6) is represented as power law:

$$\sigma = E_T^n \quad (4)$$

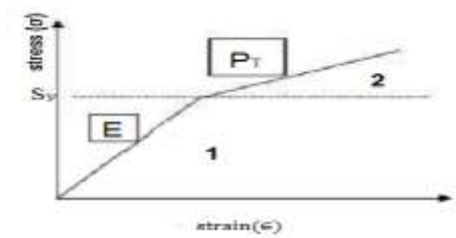

Fig..6 Stress-Strain Curve With Strain Hardening

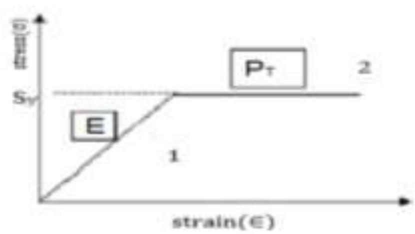

Fig.7 Stress-Strain Curve Without Strain Hardening

Plastic materials with strain hardening necessitate increasingly higher stresses to result in further plastic deformation. Since the strain hardening for highest strength engineering materials is considerably less than 0.2, the increased complexity introduced by considering strain hardening is generally not necessary within the partially plastic region.[9].Therefore, the material behavior model that is generally used for pressure vessel design is the elastic-perfectly plastic representation Figure (6) which assumes zero strain hardening [9]. From Figure (7), it can be deduced that the material has a lower load carrying capacity than one in which strain hardening is present Figure(6). In the simplified model, the proportional limits (S_Y) as well as the ultimate strength are identical. A thick-walled pressure vessel would be fully plastic when the stress in the full wall cross-section reaches the yield stress. The governing equations for formulating stress distribution for the elastic-plastic region have been derived by considering the power law:

(Eq. $\sigma = E_T^n$) based on modified Von Mises yield criterion for axially symmetric thick-walled pressure vessel.[1]. Consider a plain strain thick-walled cylinder subjected to internal pressure (p_i). When the pressure is large enough, the cylinder begins to yield from the inner surface at ($r = r_i$).

- **Failure Theories**

The distortion energy theory is a failure theory that is used to predict the failure of a tough material. It is based on the assumption that the proportion of energy that causes a component to change shape is a crucial factor in relation to the material stress. An equivalent stress is calculated from the two principal stresses σ_1 , σ_3 the distortion energy theory is used in the dimensioning of work pieces made of tough materials. Typical applications include shafts and structural steelwork. In traditional structural design, which uses strength-of material approach, the stress analysis alone cannot be able to predict The failure of a structural component.[6]. Therefore, various failure theories have been derived to combine and measure the induced stresses against the potential failure mode.

The most commonly used theories of failure are.[9]:

- Maximum principal stress theory (Rankine's failure theory)
- Maximum shear stress theory (Tresca's failure theory)
- Maximum distortion energy theory (Von Mises failure theory)

Tresca's failure theory is a historical reference standard Von Mises failure theory is actually provides realistic, classical and more accurate results and the difference between them is about 15%According to the maximum principal stress theory, failure occurs when one of the three principal stresses reaches a stress value of elastic limit as determined from a uniaxial tension test. This theory is meaningful for brittle fracture situations.[9]. The maximum shear stress theory states that, the maximum shear equals the shear stress at the elastic limit as determined from the uni-axial tension test .[9]. For a tri-axial stress system, the maximum shear stress is given as

$$\tau_{max}=\frac{\sigma_1-\sigma_3}{2}=\frac{S_Y}{2} \quad (6)$$

The Von Mises stress is:

$$\sigma'=(\sigma_1^2-\sigma_1\sigma_3+\sigma_3^2)^{1/2} \quad (7)$$

The results that obtain by ANSYS and that obtain theoretical presented in sections (5).

3. Problem Statement

Pressure vessels are commonly used for large industrial and commercial applications such as storage, filtration, and softening purposes. Pressure vessels usually bear pressure and thermal loadings namely thermo- mechanical loadings and experiences expansion loads due to change in temperature. The simulation used thick-walled cylinders with wall thickness 11.975mm at the middle of the motor case and 16 mm at both ends, having 2215 mm long and outer diameter of 267 mm and internal diameter 235mm. In this study, design and analysis are performed using commercial code to compare the stresses between different geometries. The structural design of the pressure vessel is also optimized to accommodate thermal as well structural loads. Von-mises stress, hoop stress, and deformation are plotted for all case studies.

4. Software Analysis

4.1 Designing of Modeling by ANSYS Software:

In this study, the investigated thick-walled pressure vessel is presumably made of steel 38HN3MFA, which has been increasingly applied to the design and manufacture of pressure vessel and piping .[4]

Table (1) Mechanical Properties For Steel 38HN3MFA [10]

Property	Value	Unit
Young's Modulus	$210*10^3$	MPa
Tensile strength	365	MPa
Yield strength	560	MPa
Poisson's ratio	0.3	MPa
Bulk modulus	$1.75*10^5$	MPa
Shear modulus	$8.0769*10^4$	MPa

Thick-walled pressure vessel geometrical size, were modeled in ANSYS. The first case to be modeled was that of a pressure vessel with inner radius to wall thickness ratio whose 3D section model is as show in Figure (8). Boundary conditions were applied on the surfaces clearly marked in Figure (9) by blue color. This was done by simply defining symmetry boundary conditions on the relevant areas. And pressure was applied on the internal surfaces clearly marked in Figure (10) by red color. The model was meshed using body sizing method mechanical physics preference, adaptive size function, medium smoothing, 682334 nodes and 138510 elements show in figure (11).

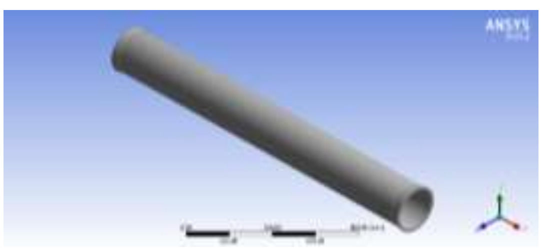

Fig.8 Model Section

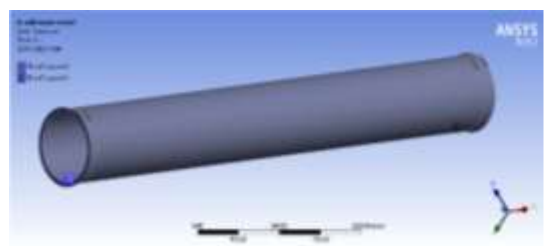

Fig.9 Fixed Support Model

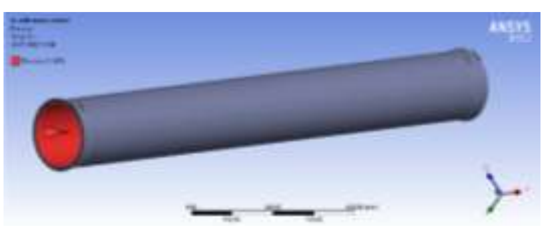

Fig.10 Pressure On Model

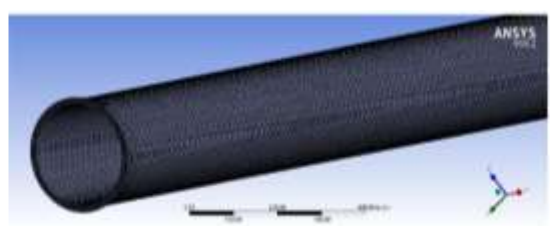

Fig.11 Meshed Model

4.2 Elastic Analysis Of Thick Walled Half Cylinder

FE Structural elastic analysis of a thick-walled pressure vessel the same geometrical size deal with in the elastic analysis of a thick-walled pressure vessel that have different diameters at both sides. Figure (12).

Shows the model with boundary conditions fixed at the two sides of the cylinder. Figure (13) illustrates the areas where boundary conditions were applied. Figure (14) illustrates the areas where pressure was applied. The same element type and element edge length as used before were maintained for this analysis. 24921 nodes and 127384 elements show in Figure (15).

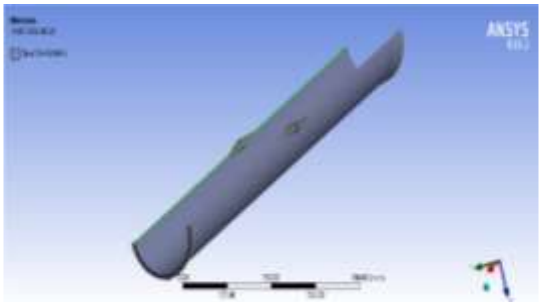

Fig.12 Model Section

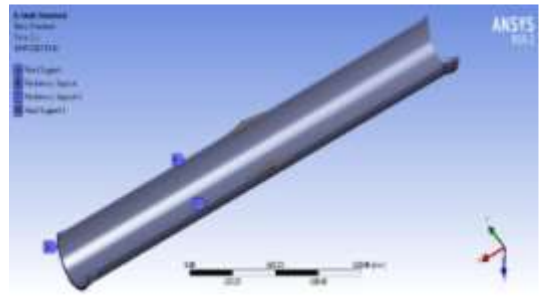

Fig.13 Fixed Support Model

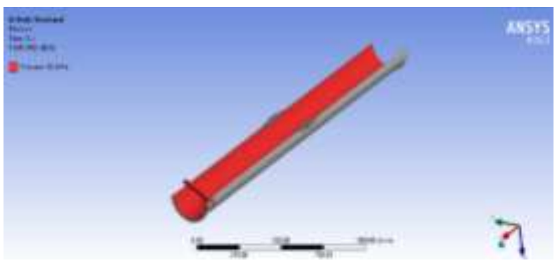

Fig.14 Pressure On Model

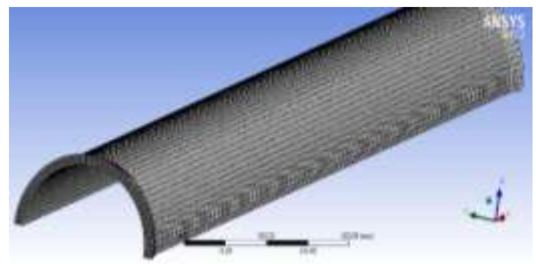

Fig.15 Meshed Model

5. Results And Discussions

5.1 Results and Discussion

The finite element analysis results obtained were compared with some calculated results in order to verify them. It is important to compare FEA results with appropriate theoretical results whenever possible so as to validate the results. However, FEA is presumably applied because a theoretical solution is not available, especially for complicated problems. In such cases, experimental results may be sought to validate the FEA results.

5.2 Elastic Analysis Results For Thick Walled Pressure Vessel (Case Study)

Figure (16) shows the effects of Von Mises stress in the pressure vessel wall applying internal pressure 150 Bar., where table (2) illustrates the FEA results for Von Mises stress acts on the pressure vessel using different pressures by using ANSYS software. Figure (17), it can be seen that the relation between the internal pressures varying from 50 Bar to 350 Bar and the Von-Mises stress.

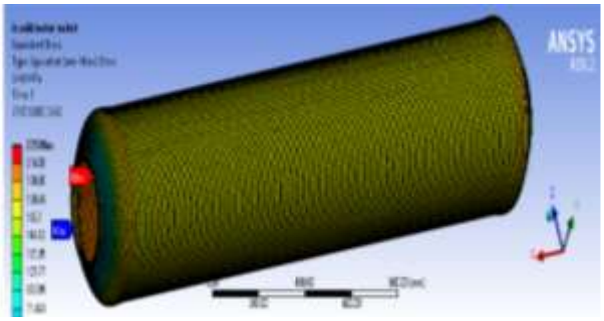

Fig.16 Von Mises Stress Results At Internal Pressure Of 150 Bar

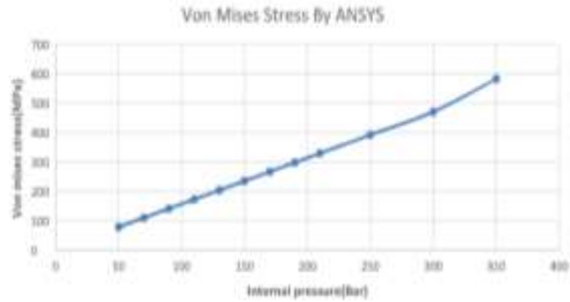

Fig.17 Von Mises Stress Results At Different Pressure

Table (2): Von Mises Stresses Obtained By ANSYS Software

Pressure (Bar)	Von Mises Stress (MPa) (ANSYS)
50	78.334
70	109.67
90	141
110	172.33
130	203.67
150	235
170	266.33
190	297.67
210	329
250	391.67
300	470
350	581.75

5.3 Elastic Analysis Results For Thick Walled Pressure Vessel [Uniform Pressure Vessel (The Same Thickness Of Cylinder Wall Along Pressure Vessel)]

Figure (18) show the effects of Von Mises stress in the pressure vessel wall applying internal pressure of 50 Bar. Table (3) illustrates the FEA results for Von Mises stress in the pressure vessel using different pressures by using ANSYS software and calculation. Table (3) presents the results of Von-Mises stress obtained from ANSYS software and calculated stresses at different internal pressure varying from 50 Bars to 350 Bars. These results are summarized graphically in figure (19). Figure (20) shows the stress distribution for regular cylinder without change in diameter (no discontinuity), the maximum stress was 60.8Mpa at 50 Bar, where the maximum stress for the same cylinder with different diameter at the two ends at the same internal pressure 50 Bar was78.3Mpa as shown in figure(16). Figure (21) show the effects for total deformation in the pressure vessel wall, where the maximum displacement is in the internal surface of the vessel at the place of radius change (discontinuity), and table (4) illustrates the FEA results for total deformation in the pressure vessel using different pressure by ANSYS software.

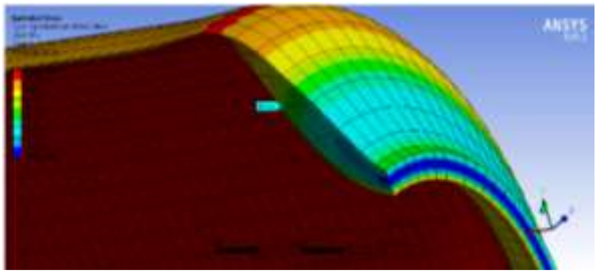

Fig.18 Von Mises Stress Results at 50 Bar

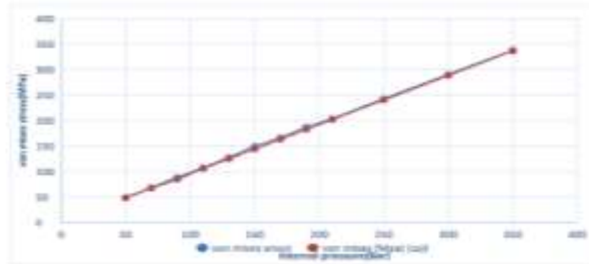

Fig. (19) Von Mises Stresses By ANSYS Software And Calculated Stress

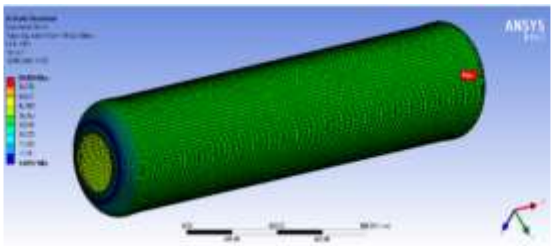

Fig..20 Von Mises Stress For Regular Pressure Vessel at 50 Bar

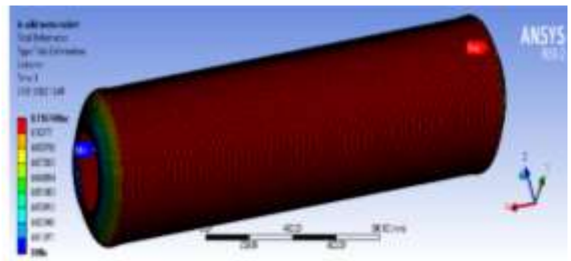

Fig. .21 Resultant Displacement Model At 150 Bar

Table (3) Von Mises Stresses By ANSYS Software And Calculated Stress (same thickness pressure vessel)

Pressure (Bar)	Von Mises Stress (MPa) (ANSYS)	Von Mises Stress (MPa) (Cal)	Percentage %
50	49.415	48.013	2.837%
70	68.669	67.21	2.124%
90	88.451	84.813	4.113%
110	107.562	105.63	1.796%
130	127.652	124.832	2.209%
150	149.354	144.037	3.5%
170	166.327	163.242	1.854%
190	187.256	182.443	2.574%
210	203.569	201.653	0.941%
250	242.898	240.063	1.167%
300	290.542	288.084	0.846%
350	338.217	336.09	0.629%

Table (4) Total Deformation Of Pressure Vessel

Pressure (Bar)	Displacement (mm)
50	0.0389
70	0.05445
90	0.07004
110	0.08561
130	0.10117
150	0.11674
170	0.1323
190	0.14784
210	0.16343
250	0.19456
300	0.23347
350	0.27239

Sometimes, it may be necessary to know the exact values of induced stress at various specific points on a component. This can be enabled by using the “Probe Results” command through selecting the desired component and then picking the appropriate node on the specified point or region. It may be helpful to zoom in on the particular region. Figure (22) shows the values of Von Mises stress at internal pressure (50Bar) at various chosen points on the wall thickness of the pressure vessel. And table (5) illustrates the FEA results for the values of Von Mises stresses at various thicknesses by ANSYS program and calculation. Table (6) illustrates the hoop and radial stresses at internal pressure 150 Bar, results at different thickness Figure (23) and (24) shows the hoop and radial stresses from the tables.

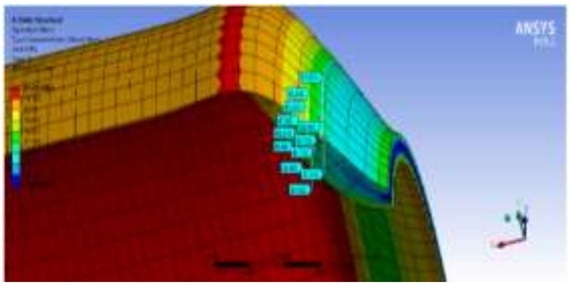

Fig..22 Von Mises Stresses For Various Points Along The Wall Thickness

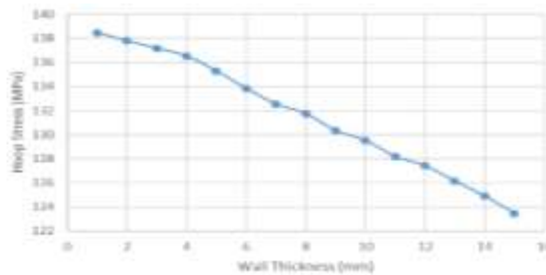

Fig.23 Hoop Stresses For Various Thickness Using Formula

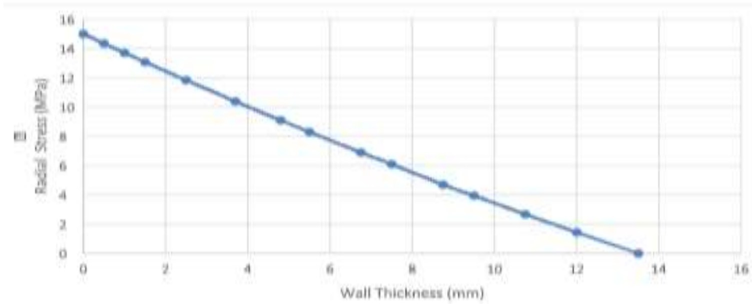

Fig.24 Radial Stresses For Various Thicknesses Using Formula

Table (5) Von Von Mises Stresses For Various Thickness By ANSYS And Calculated Stress

Thickness (mm)	Von Mises Stress (MPa) (ANSYS)	Hoop Stresses (MPa)	Radial Stress(MPa)	Von Mises Stress (MPa) (Cal)	Percentage %
0	49.102	46.1543	-5	48.8466	0.523 %
0.5	48.557	45.9380	-4.7837	48.5071	0.103 %
1	48.034	45.7244	-4.5701	48.1724	0.287 %
1.5	47.551	45.5135	-4.3592	47.8424	0.609 %
2.5	47.121	45.0997	-3.9453	47.1962	0.159 %
3.7	46.319	44.6165	-3.4622	46.4445	0.270 %
4.8	45.334	44.1862	-3.0317	45.7772	0.968 %
5.5	44.707	43.9180	-2.7637	45.3631	1.446 %
6.75	44.05	43.4508	-2.2964	44.6433	1.329 %
7.5	43.129	43.1771	-2.0228	44.2232	2.474 %
8.75	41.811	42.7318	-1.5775	43.5420	3.975 %

9.5	40.486	42.4709	-1.3166	43.1443	6.161 %
10.75	39.319	42.0462	-0.8919	42.4992	7.483 %
12	38.333	41.6337	-0.4794	41.8755	8.459 %
13.5	37.252	41.154	0	41.1543	9.482 %

Table (6) Hoop Stresses For Various Thickness Using Formula

Thickness(mm)	Hoop Stresses (MPa)	Radial Stress(MPa)
0	138.463	-15
0.5	137.814	-14.35
1	137.173	-13.71
1.5	136.540	-13.07
2.5	135.299	-11.83
3.7	133.845	-10.38
4.8	132.557	-9.095
5.5	131.754	-8.292
6.75	130.352	-6.889
7.5	129.531	-6.068
8.75	128.195	-4.732
9.5	127.414	-3.949
10.75	126.139	-2.675
12	124.901	-1.438
13.5	123.463	0

5.4 Plastic analysis results for thick walled pressure vessel

If the internal pressure is increased to values greater than the initial yielding pressure, plastic deformation initiates at the inner surface and proceeds outward through the cylinder wall. The interface between the plastically deformed material and the elastic material will eventually reach the outer surface at a value of pressure that is known as the ‘‘full plastic flow pressure. FE plastic analysis - plastic region of different depths from the FEA solution, Figure (25) shows the value of yield strength at the thickness of a specific wall of the pressure vessel at internal pressure 350 Bar. Table (7) illustrates the yield stress distribution across the wall thickness where Figure (26) shows the yield stress distribution along the wall thickness.

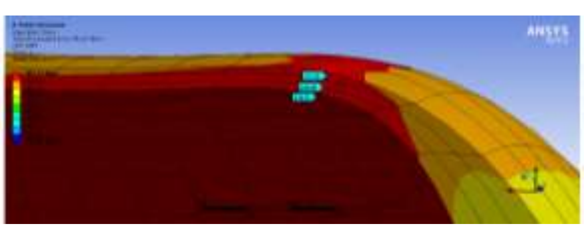

Fig.25 Value Of Yield Strength At Certain Thickness

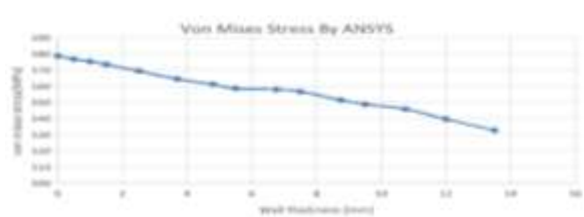

Fig.26 Yield Stress Versus Wall Thickness At Different Pressures

Table (7) Von Mises Stress For Various Thicknesses At Internal Pressure 350 Bar Obtained By ANSYS Software

Thickness(mm)	Von Mises Stress (MPa) (ANSYS)
0	578.55
0.5	576.53
1	575.12
1.5	573.25
2.5	569.256
3.7	564.25
4.8	560.96
5.5	558.46
6.75	557.76
7.5	556.45
8.75	551.26
9.5	548.652
10.75	545.65
12	539.456
13.5	532.56

6. Conclusion

Thick cylinders were subjected to different high internal pressures andanalysis was also done and various conclusions were drawn from data's and graphs.

- In elastic stress case the hoop stress and Von Mises stress distribution varies in a similar manner.
- Increasing the internal pressure to values greater than the initial yielding pressure, plastic deformation initiates at the inner surface and proceeds outward through the cylinder wall.
- The interface between the plastically deformed material and the elastic material will eventually reach the outer surface at a value of pressure that is known as the ‘‘full plastic pressure.
- Local structural discontinuities are sources of stress or strain intensification that affect only a small volume of material and do not have a significant effect upon the overall stress pattern. They usually produce peak stresses.
- The results show that the Von Mises stresses was high for discontinues shape of motor case compared by the uniform motor case (constant thickness).
- The plastic deformation starts from the inner surface of motor case and goes towards the outer diameter as the internal pressure increases.
- The plastic deformation occurred at pressure of 350 Bar, and exceeds the yield strength of the material used.
- The results show a good agreement between the results obtained by ANSYS software and the calculation using the formula of Von Mises stresses.
- The maximum values of Von Mises stress were noticed at the point of change of the external diameter of motor case at both ends.

7. Reference

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