Solving (3+1)-dimensional Jimpo-Miwa equations using Riccati mapping equation method

Abdulmalik A. Altwaty [1], Emad Mikael[2] and Bader R. K. Masry[2]
[1] Department of Mathematics, Faculty of Science, University of Benghazi, AL KUFRA, Libya
[2] Department of Mathematics, Faculty of Science, University of Tobruk, Tobruk, Libya

Abstract
Several new exact solutions of the $(3+1)$-dimensional Jimbo-Miwa equation and its two extended forms have been successfully obtained by using the Riccati mapping equation method. Although some of the dispersion relations are the same as in [17] our solutions consedered new .
Key words: *Riccati equation mapping method,* $(3+1)$*-dimensional Jimpo-Miwa equations, Exact solutions*.

الملخص الكثير من الحلول التامة الجديدة لمعادلة جامبو–ميوا ذات البعد $(3+1)$ وصيغتاها الموسعتان تم الحصول عليها بنجاح باستخدام طريقة معادلة ريكاتي. بالرغم من ان علاقات التشتت تعتبر نفسها في [17] الا ان حلولنا تعتبر جديدة.

الكلمات المفتاحية: طريقة معادلة ريكاتي، معالادت جمبو–ميوا ذات البعد (3+1)، حلول تامة

1 Introduction

Various powerful methods have been used to refine various types of solutions for nonlinear partial differential equations models that are used to explain a broad variety of physical phenomena. . Some of these methods are $\tan(F(\frac{\zeta}{2}))$-expansion method [1], the sine-cosine function method [2], the variational iteration method [3,4], Riccati mapping equation method $[5-7]$, the exp-function method [8,9], the generalized and the improved generalized tanh-coth method [10,11], the new modified simple equation method [12], the simplified Hirota's direct method [13,14], tanh method, sine-Gordon expansion method, and the extended sinh-Gordon equation expansion method [15,16]. Wazwaz [17] has obtained multiple soliton solutions of distinct physical structures for $(3+1)$-dimensional Jimbo-Miwa and its two extended equations by using simplified Hirota's method. Many other solutions have been found by several researchers $[18-20]$.

In this article, we use Riccati mapping equation method to extract many new solutions of both the $(3+1)$-dimensional Jimbo-Miwa equation and its two extended forms as well. The dispersion relations are found to be distinct for each equation.

2 Description of the Riccati equation mapping method

Consider a given nonlinear partial differential equation with the independent variable $X=(t,x_1,x_2,\dots,x_n)$ and the dependent variable $u(X)$ in the following form;

$$W(u,u_t,u_{x_i},u_{x_ix_i},u_{x_ix_j},\dots)=0, \tag{1}$$

where W is a general polynomial function of its arguments. In order to solve Eq.(1) by using the proposed method, the following main steps are given;

Step 1: Substituting

$$u = u(\zeta), \quad \zeta = k\,x + r\,y + s\,z - c\,t, \tag{2}$$

into (1) yields an ordinary differential equation in ζ of the form

$$E(u, u'(\zeta), u''(\zeta), u'''(\zeta), \dots) = 0, \tag{3}$$

where E is a general polynomial.

Step 2: Assuming that (3) has the formal solution

$$u(\zeta) = a_0 + \sum_{i=1}^{m} \left(a_i\,\phi^i(\zeta) + b_i\,\phi^{i-1}(\zeta)\sqrt{(\sigma + \phi^2(\zeta))}\right), \tag{4}$$

with ϕ satisfies the Riccati equation;

$$\phi'(\zeta) = \sigma + \phi^2(\zeta), \tag{5}$$

where σ, a_0, a_i, and b_i are constants to be determined later, while m is a positive integer, which is called the balance number.

Step 3: Determining the positive integer m by balancing the highest order derivatives and the nonlinear terms in Eq. (3).

Step 4: Substituting (4) along with Eq. (5) into (3) and collecting all the coefficients of

$\phi^n\left(\sqrt{\sigma + \phi^2(\zeta)}\right)^m$, $n = 0, \pm1, \pm2, \pm3, \dots$; $m = 0, \pm1$ then setting each coefficient to zero,

a set of algebraic equations is obtained for a_0, a_i, b_i, and c.

Step 5: Solving the system of algebraic equations in **Step4** using Matlab or Mathematica software to find the values of a_0, a_i, b_i, and c.

Step 6: As Eq.(5) possesses the solutions;

$$\phi(\zeta) = \begin{cases} -\sqrt{-\sigma}\ \tanh(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ -\sqrt{-\sigma}\ \coth(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ \sqrt{\sigma}\ \tan(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ \sqrt{\sigma}\ \cot(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ -\frac{1}{\zeta} & ,\sigma = 0, \end{cases} \tag{6}$$

substituting a_0, a_i, b_i, and (6) into (4) to obtain the exact solutions of Eq. (1).

3. Applications

In this section, new exact solutions of the nonlinear $(3+1)$-dimensional Jimpo-Miwa equation and its two extracted forms have been developed by using the described Riccati equation mapping method. Without loss of generality we will assume through out this paper that the integration constants are set to be zeros.

3.1 The exact solutions of $(3+1)$-dimensional Jimpo-Miwa equation:

$$u_{xxxy}+3\,u_y u_{xx}+3\,u_x u_{xy}+2\,u_{yt}-3\,u_{xz}=0. \qquad (7)$$

Using the wave transformations $u=u(\zeta)$, $\zeta=k\,x+r\,y+s\,z-c\,t$, where k, r, s and c are constants in equation (7) and integrating we get

$$k^3\,r\,u'''+3\,k^2\,r\,(u')^2-(2\,r\,c+3\,k\,s)u'=0. \qquad (8)$$

Balancing u''' with $(u')^2$ we get $m=1$. From (4) we have

$$u(\zeta)=a_0+a_1\,\phi(\zeta)+b_1\sqrt{\sigma+\phi^2(\zeta)}. \qquad (9)$$

Substituting (9) and (5) into equation (8) and collecting all the coefficients of $\phi^n\left(\sqrt{\sigma+\phi^2(\zeta)}\right)^m$ and setting them equal to zero we obtain

Case 1: $a_0=0$, $a_1=-k$, $b_1=-k$, and $c=-(\frac{k^3\,r\,\sigma+3\,k\,s}{2\,r})$. The exact solution in this case is

$$u(\zeta)=\begin{cases} k\sqrt{-\sigma}\,\tanh(\sqrt{-\sigma}\,\zeta)-k\sqrt{\sigma}\,sech(\sqrt{-\sigma}\,\zeta) & ,\sigma<0,\\ k\sqrt{-\sigma}\,\coth(\sqrt{-\sigma}\,\zeta)+k\sqrt{\sigma}\,csch(\sqrt{-\sigma}\,\zeta) & ,\sigma<0,\\ -k\sqrt{\sigma}\,\tan(\sqrt{\sigma}\,\zeta)-k\sqrt{\sigma}\,sec(\sqrt{\sigma}\,\zeta) & ,\sigma>0,\\ -k\sqrt{\sigma}\,\cot(\sqrt{\sigma}\,\zeta)-k\sqrt{\sigma}\,csc(\sqrt{\sigma}\,\zeta) & ,\sigma>0,\\ 0 & ,\sigma=0, \end{cases} \qquad (10)$$

Remark: For $\sigma=-1$ the dispersion relation is completely agree with [17]

Case 2: $a_0=0$, $a_1=-2k$, $b_1=0$, $c=-(\frac{4\,k^3\,r\,\sigma+3\,k\,s}{2\,r})$ and the exact solution is

$$u(\zeta)=\begin{cases} 2\,k\sqrt{-\sigma}\,\tanh(\sqrt{-\sigma}\,\zeta) & ,\sigma<0,\\ 2\,k\sqrt{-\sigma}\,\coth(\sqrt{-\sigma}\,\zeta) & ,\sigma<0,\\ -2\,k\sqrt{\sigma}\,\tan(\sqrt{\sigma}\,\zeta) & ,\sigma>0,\\ -2\,k\sqrt{\sigma}\,\cot(\sqrt{\sigma}\,\zeta) & ,\sigma>0,\\ \frac{2k}{\zeta} & ,\sigma=0, \end{cases} \qquad (11)$$

Remark: For $\sigma=-\frac{1}{4}$ the dispersion relation is completely agree with [17]

Case 3: $a_0=0$, $a_1=-k$, $b_1=k$, $c=-(\frac{k^3\,r\,\sigma+3\,k\,s}{2\,r})$ and the exact solution is

$$u(\zeta)=\begin{cases} k\sqrt{-\sigma}\,\tanh(\sqrt{-\sigma}\,\zeta)+k\sqrt{\sigma}\,sech(\sqrt{-\sigma}\,\zeta) & ,\sigma<0,\\ k\sqrt{-\sigma}\,\coth(\sqrt{-\sigma}\,\zeta)-k\sqrt{\sigma}\,csch(\sqrt{-\sigma}\,\zeta) & ,\sigma<0,\\ -k\sqrt{\sigma}\,\tan(\sqrt{\sigma}\,\zeta)+k\sqrt{\sigma}\,sec(\sqrt{\sigma}\,\zeta) & ,\sigma>0,\\ -k\sqrt{\sigma}\,\cot(\sqrt{\sigma}\,\zeta)+k\sqrt{\sigma}\,csc(\sqrt{\sigma}\,\zeta) & ,\sigma>0,\\ \frac{2k}{\zeta} & ,\sigma=0, \end{cases} \qquad (12)$$

3.2 The exact solutions of the first extended $(3+1)$-dimensional Jimbo-Miwa equation:

$$u_{xxxy} + 3\, u_y u_{xx} + 3\, u_x u_{xy} + 2\, u_{yt} - 3\,(u_{xz} + u_{yz} + u_{zz}) = 0. \tag{13}$$

Substituting the wave transformation $u = u(\zeta)$, $\zeta = k\,x + r\,y + s\,z - c\,t$ into (13) and integrating we get

$$k^3\, r\, u''' + 3\, k^2\, r\,(u')^2 - (3\,k\,s + 3\,s\,r + 3\,s^2 + 2\,r\,c)u' = 0. \tag{14}$$

Balancing u''' with $(u')^2$ we get $m = 1$. From (4) we have

$$u(\zeta) = a_0 + a_1\,\phi(\zeta) + b_1\,\sqrt{\sigma + \phi^2(\zeta)}. \tag{15}$$

Substituting (5) and (15) into equation (14) and collecting all the coefficients of $\phi^n\left(\sqrt{\sigma + \phi^2(\zeta)}\right)^m$ and setting them equal to zero we obtain

Case 1: $a_0 = 0$, $a_1 = -k$, $b_1 = k$, and $c = -(\frac{k^3\, r\,\sigma + 3\,k\,s + 3\,s^2 + 3\,r\,s}{2\,r})$. The exact solution in this case is

$$u(\zeta) = \begin{cases} k\sqrt{-\sigma}\ \tanh(\sqrt{-\sigma}\,\zeta) + k\,\sqrt{\sigma}\,sech(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ k\sqrt{-\sigma}\ \coth(\sqrt{-\sigma}\,\zeta) - k\,\sqrt{\sigma}\ csch(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ -k\sqrt{\sigma}\ \tan(\sqrt{\sigma}\,\zeta) + k\,\sqrt{\sigma}\ sec(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ -k\sqrt{\sigma}\ \cot(\sqrt{\sigma}\,\zeta) + k\,\sqrt{\sigma}\ csc(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ \frac{2\,k}{\zeta} & ,\sigma = 0, \end{cases} \tag{16}$$

Remark: For $\sigma = -1$ the dispersion relation is completely agree with [17]

Case 2: $a_0 = 0$, $a_1 = -k$, $b_1 = -k$, $c = -(\frac{k^3\, r\,\sigma + 3\,k\,s + 3\,s^2 + 3\,r\,s}{2\,r})$ and the exact solution is

$$u(\zeta) = \begin{cases} k\sqrt{-\sigma}\ \tanh(\sqrt{-\sigma}\,\zeta) - k\,\sqrt{\sigma}\,sech(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ k\sqrt{-\sigma}\ \coth(\sqrt{-\sigma}\,\zeta) + k\,\sqrt{\sigma}\ csch(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ -k\sqrt{\sigma}\ \tan(\sqrt{\sigma}\,\zeta) - k\,\sqrt{\sigma}\ sec(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ -k\sqrt{\sigma}\ \cot(\sqrt{\sigma}\,\zeta) - k\,\sqrt{\sigma}\ csc(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ 0 & ,\sigma = 0, \end{cases} \tag{17}$$

Case 3: $a_0 = 0$, $a_1 = -2k$, $b_1 = 0$, $c = -(\frac{4\,k^3\, r\,\sigma + 3\,k\,s + 3\,s^2 + 3\,r\,s}{2\,r})$ and the exact solution is

$$u(\zeta) = \begin{cases} 2k\sqrt{-\sigma}\ \tanh(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ 2k\sqrt{-\sigma}\ \coth(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ -2k\sqrt{\sigma}\ \tan(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ -2k\sqrt{\sigma}\ \cot(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ \frac{2k}{\zeta} & ,\sigma = 0, \end{cases} \tag{18}$$

Remark: For $\sigma = -\frac{1}{4}$ the dispersion relation is completely agree with [17]

3.3 The exact solutions of the second extended $(3+1)$-dimensional Jimbo-Miwa equation:

$$u_{xxxy} + 3\,u_y u_{xx} + 3\,u_x u_{xy} + 2\,(u_{xt} + u_{yt} + u_{zt}) - 3\,u_{xz} = 0, \qquad (19)$$

Substituting the wave transformation $u = u(\zeta)$, $\zeta = k\,x + ry + s\,z - c\,t$ into (19) and integrating we get

$$k^3\,r\,u''' + 3\,k^2\,r\,(u')^2 - (2\,k\,c + 2\,s\,c + 2\,r\,c + 3\,k\,s)u' = 0. \qquad (20)$$

Balancing u''' with $(u')^2$ we get $m = 1$. From (4) we have

$$u(\zeta) = a_0 + a_1\,\phi(\zeta) + b_1\,\sqrt{(\sigma + \phi^2(\zeta))}. \qquad (21)$$

Substituting (5) and (21) into (20) and collecting all the coefficients of $\phi^n\left(\sqrt{\sigma + \phi^2(\zeta)}\right)^m$ and setting them equal to zero we obtain

Case 1: $a_0 = 0$, $a_1 = -k$, $b_1 = k$, and $c = -(\frac{k^3\,r\,\sigma + 3\,k\,s}{2\,(k+r+s)})$. The exact solution in this case is

$$u(\zeta) = \begin{cases} k\sqrt{-\sigma}\ \tanh(\sqrt{-\sigma}\,\zeta) + k\,\sqrt{\sigma}\,sech(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ k\sqrt{-\sigma}\ \coth(\sqrt{-\sigma}\,\zeta) - k\,\sqrt{\sigma}\ csch(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ -k\sqrt{\sigma}\ \tan(\sqrt{\sigma}\,\zeta) + k\,\sqrt{\sigma}\ sec(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ -k\sqrt{\sigma}\ \cot(\sqrt{\sigma}\,\zeta) + k\,\sqrt{\sigma}\ csc(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ \frac{2\,k}{\zeta} & ,\sigma = 0, \end{cases} \qquad (22)$$

Remark: For $\sigma = -1$ the dispersion relation is completely agree with [17]

Case 2: $a_0 = 0$, $a_1 = -k$, $b_1 = -k$, $c = -(\frac{k^3\,r\,\sigma + 3\,k\,s}{2\,(k+r+s)})$ and the exact solution is

$$u(\zeta) = \begin{cases} k\sqrt{-\sigma}\ \tanh(\sqrt{-\sigma}\,\zeta) - k\,\sqrt{\sigma}\,sech(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ k\sqrt{-\sigma}\ \coth(\sqrt{-\sigma}\,\zeta) + k\,\sqrt{\sigma}\ csch(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ -k\sqrt{\sigma}\ \tan(\sqrt{\sigma}\,\zeta) - k\,\sqrt{\sigma}\ sec(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ -k\sqrt{\sigma}\ \cot(\sqrt{\sigma}\,\zeta) - k\,\sqrt{\sigma}\ csc(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ 0 & ,\sigma = 0, \end{cases} \qquad (23)$$

Case 3: $a_0 = 0$, $a_1 = -2k$, $b_1 = 0$, $c = -(\frac{4\,k^3\,r\,\sigma + 3\,k\,s}{2\,(k+r+s)})$ and the exact solution is

$$u(\zeta) = \begin{cases} 2k\sqrt{-\sigma}\ \tanh(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ 2k\sqrt{-\sigma}\ \coth(\sqrt{-\sigma}\,\zeta) & ,\sigma < 0, \\ -2k\sqrt{\sigma}\ \tan(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ -2k\sqrt{\sigma}\ \cot(\sqrt{\sigma}\,\zeta) & ,\sigma > 0, \\ \frac{2k}{\zeta} & ,\sigma = 0, \end{cases} \qquad (24)$$

Remark: For $\sigma = -\frac{1}{4}$ the dispersion relation is completely agree with [17]

4 Conclusion

In this work, the Riccati mapping equation method has been applied to solve the $(3+1)$-dimensional Jimbo-Miwa equation as well as its two extended forms. We obtained various types of solutions as trigonometric and rational functions. It is verified that these solutions are satisfying their corresponding equations. The obtained results have been compared with Wazwaz [17] and the observation was that our results are new with different solution structures.

References

[1] I. E. Inan, $\tan(F(\frac{\zeta}{2})$-Expansion method for traveling wave solutions of the variant Bussinesq equations, Karaelmas Fen ve Math. Derg., Vol. 7(1), 201-206, 2017.

[2] X. Liang, Z. Cai, M. Wang, X. Zhao, H. Chen, C. L. Chaotic, oppositional sine–cosine method for solving global optimization problems, Engineering with Computers. (2020)

[3] A. M. Wazwaz, Optical bright and dark soliton solutions for coupled nonlinear Schrödinger (CNLS) equations by the variational iteration method, Optik-International Journal for Light and Electron Optics 207 (2020) 164457.

[4] J. He, Variational iteration method: Some recent results and new interpretations, J. Comput. Appl. Math., Vol. 207(1), (2007) 3-17.

[5] E. M. E. Zayed, S. A H. Ibrahim, The Riccati equation mapping method for solving nonlinear partial differential equations in mathematical physics, Walailak J Sci and Tech., Vol. 11(7), (2014) 621-632.

[6] E. M.E. Zayed, Y. A. Amer and R. M.A. Shohib, The improved generalized Riccati equation mapping method and its application for solving a nonlinear partial differential equation (PDE) describing the dynamics of ionic currents along microtubules, Academic Journals, Vol. 9(8), (2014) 238-248.

[7] S. Guo, L. Mei, Y. Zhou, C. Li, The extended Riccati equation mapping method for variable-coefficient diffusion-reaction and mKdV equations, Applied mathematics and computation, Vol. 217, (2011) 6264-6272.

[8] X. Wu, J. He, Solitary solutions, periodic solutions and compacton-like solutions using Exp-function method, Comput. Math. Appl, Vol. 54, (2007) 966-986.

[9] T. Ozis, I. Aslan, Exact and explicit solutions to the (3+1)-dimensional Jimbo-Miwa equation via the Exp-function method, Physics Letters A, Vol. 372, (2008) 7011-7015.

[10] C. Sierra, A. H. Salas, The generalized tanh-coth method to special types of the fifth-order KdV equation, Applied Mathematics and Computation, Vol. 203, (2008) 873-880.

[11] M. Torvattanabun, J. Simmapim, D. Saennuad, and T. Somaumchan, The improved generalized tanh-coth method applied to sixth-order solitary wave equation, Hindawi Journal of Mathematics, Vol. 2017, Article ID 8751097, 11 pages.

[12] A. Irshad, S. Mohyud-Din, Naveed Ahmed, Umar Khan, A new modification in simple equation method and its applications on nonlinear equations of physical nature, Results in Physics, Vol. 7, (2017) 4232-4240.

[13] A. M. Wazwaz, The simplified Hirota's method for studying three extended higher-order KdV-type equations, Journal of Ocean Engineering and Science, Vol. 1, (2016) 181-185.

[14] R. Hirota, The direct method in soliton theory, Iwanami Shoten (1992) [Japanese], English transl. Cambridge University Press, Cambridge (2004).

[15] A. M. Wazwaz, Compactons and solitary wave solutions for the Boussinesq wave equation and its generalized form, Appl. Math. Comput, Vol. 182, (2006) 529-535.

[16] A.M. Wazwaz, New hyperbolic schemes for reliable treatment of Boussinesq equation, Phys. Lett. A, Vol. 358, (2006) 409-413.

[17] A. M. Wazwaz, Multiple-soliton solutions for extended $(3+1)$-dimensional Jimbo-Miwa equations, Applied mathematics letters, Vol. 64, (2017) 21-26.

[18] K. Ali , R.I. Nuruddeen , A. R. Hadhoud, New exact solitary wave solutions for the extended $(3+1)$-dimensional Jimbo-Miwa equations, Results in Physics, Vol. 9, (2018) 12-16.

[19] T. Su and H. Dai, Theta function solutions of the $(3+1)$-dimensional Jimbo-Miwa equation. Hindawi. Mathematical problems in engineering. Volume 2017, Article ID 2924947, 9 pages

[20] Y. Zhang, S. Sun, and H. Dong, Hybrid solutions of $(3+1)$-dimensional Jimbo-Miwa equation, Hindawi. Mathematical Problems in Engineering. Volume 2017, Article ID 5453941, 15 pages