Semi-connected Spaces in Neutrosophic Crisp Set Theory

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Abstract

In this study, we introduce and study concept of semi-connected space in neutrosophic crisp topological space. Several properties, functions properties of neutrosophic crisp semi-connected spaces are studied. In addition to these, we introduce and study the definition of neutrosophic crisp semi-$lindel\ddot{o}ff$ spaces. We explain that neutrosophic crisp semi-connected spaces is preserved under nutrosophic crisp irresolute function and neutrosophic crisp semi-closed function with nutrosophic crisp semi-connected point inverse.

الفضاءات شبه المترابطة في نظرية المجموعات الهشة النيتروسوفكية

في هذه الدراسة قمنا بتقديم ودراسة مفهوم الفضاء شبه المترابط في الفضاء التوبولوجي الهش النيوتروسوفيكي. تمت دراسة العديد من الخصائص وخصائص الدوال للفراغات شبه المترابطة الهشة النيوتروسوفكية. وبالإضافة إلى ذلك، قمنا بتقديم ودراسة تعريف الفضاءات النيوتروسوفكية الهشة شبه الليندیلوفية. نوضح أن المساحات شبه المترابطة الهشة النيوتروسوفكية يتم الحفاظ عليها تحت دالة غير ثابتة هشة نيوتروسوفكية ودالة شبه مغلقة هشة نيوتروسوفكية مع نقطة شبه مترابطة هشة نيوتروسوفكية معكوسة.

Key Words

Neutrosophic crisp semi-connected spaces, Neutrosophic crisp semi-$lindel\ddot{o}ff$, Neutrosophic topological spaces, Neutrosophic crisp semi-open sets, Neutrosophic crisp functions.

1- Introduction and Preliminaries

In [15], Smarandache presented the concept of neutrosophic set as a new instrument to the world of Mathematic. Salama and Alblowi [9] studied the topological structure of the family of neutrosophic sets and introduced the concept of neutrosophic topological space by using membership functions, indeterminacy functions and non-membeship functions, each of which have one-to-one corresponding between the members of X and]-0, 1+
Salama et al. [9] developed neutrosophic topological spaces in 2012. Recently, Iswarya et al. [2,3,4] deliberated the notion of neutrosophic semi-open and neutrosophic semi-closed sets in 2016, neutrosophic semi-frontier in 2017 and neutrosophic semi-irresolute function in 2021. In 2021, Ahu Acikgoz et al. [1] wish to studied some concepts of connected spaces namely, neutrosophic connected and neutrosophic strongly connected.
Neutrosophic crisp sets were introduced by Salama and Smarandache in 2015. Neutrosophic topological spaces and many applications have been investigated by Salama et al. [5,8,10,11]

and[11,13]. The notions and terminologies not explained in this paper may be found in [12]. Some definitions and results which will be needed in this paper are recalled here.

In this paper we delve into the concept of neutrosophic crisp semi-connected spaces, which bridge the gap between semi-connectivity in crisp spaces and the neutrosophic framework. We examine their properties, constructions, and explore their applications in various fields.

Throughout this article, we use the acronym for the clarity of the presentation (see Table1)

Table (1) List of Short Terms

String of Words	*Acronym / Abbreviation*
Neutrosophic Open Set	NO-set
Neutrosophic Crisp Open Set	NCO-set
Neutrosophic Closed Set	NC-set
Neutrosophic Crisp Closed Set	NCC-set
Neutrosophic Topological Space	NTS
Neutrosophic Crisp Topological Space	$NCTS$
Neutrosophic Crisp Connected	NC-connected
Neutrosophic Crisp Semi-Connected	NC-Semi-connected
Neutrosophic Crisp $Lindel\ddot{o}ff$	NC-$Lindel\ddot{o}ff$
Neutrosophic Crisp Semi-$Lindel\ddot{o}ff$	NC -Semi-$Lindel\ddot{o}ff$
Neutrosophic Crisp Interior of (A)	NC-$int(A)$
Neutrosophic Crisp Closure of (A)	NC-$cl(A)$
Finite Intersection Property	FIP

Definition 1.1 [9]

Let (X, τ) be a neutrosophic topological space over X and A be a neutrosophic set over X. Then A is said to be a neutrosophic closed set iff its complement is a neutrosophic open set.

Definition 1.2 [10]

Let (X, τ) be a $NCTS$ and $A = \langle A_1, A_2, A_3 \rangle$ be a NC-set in X, then the neutrosophic crisp closure of $A(NCcl(A)$ for short) and neutrosophic crisp interior ($NCint(A)$ for short) of A are defined by

i- $NCcl(A) = \bigcap\{K: K\ is\ a\ NC-closed\ set\ in\ X\ and\ A \subseteq K\}$.

ii- $NCint(A) = \bigcup\{G: G\ is\ a\ NC-open\ set\ in\ X\ and\ G \subseteq A\}$.

It can be also shown that $NCcl(A)$ is a NC-closed set, and $NCint(A)$ is a NC-open set in X.

Definition 1.3 [2]

Let A be a neutrosophic set in a neutrosophic topological space X. Then A is said to be a neutrosophic semi-open set in X if there exists a neutrosophic open set U such that $U \subseteq A \subseteq Ncl(U)$.

Definition 1.4 [2]

Let A be a neutrosophic set in a neutrosophic topological space X. Then A is said to be a N-semi-closed set in X if there exists a neutrosophic closed set B such that $Nint(B) \subseteq A \subseteq B$.

Definition 1.5 [7]

Let (X, τ) be neutrosophic topological space and A be neutrosophic set. Then
i- The neutrosophic of A, denoted by $Nint(A)$ is the union of all neutrosophuc open subset of A. Clearly $Nint(A)$ is the biggest neutrosophic open subset of X contained in A.
ii- The neutrosophic closure of A denoted by $Ncl(A)$ is the intersection of all neutrosophic closed sets containing A. Clearly $Ncl(A)$ is the smallest neutrosophic closed set which contains A.

Definition 1.6 [12]

For any non-empty fixed set X, a neutrosophic crisp set (NC-set, for short) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$, where A_1, A_2 & A_3 are subsets of X, satisfying
$A_1 \cap A_2 = \emptyset, A_1 \cap A_3 = \emptyset$ & $A_3 \cap A_2 = \emptyset$.
Several relations and operations between NC-sets were defined in[10].

Definition 1.7 [12]

A neutrosophic crisp topology ($NCT, for\ short$) on a non-empty set X is a family τ of neutrosophic crisp subsets of X satisfying the following axioms
i- $\emptyset_N, X_N \in \tau$.
ii- $A_1 \cap A_2 \in \tau$, for any A_1 & $A_2 \in \tau$.
iii- $\bigcup A_j \in \tau$, for any $\{A_j : j \in J\} \subseteq \tau$.
In this case the pair (X, τ) is called a neutrosophic crisp topological space ($NCT, for\ short$) in X. The elements in τ are called neutrosophic crisp open sets (NC-open sets, for short) in X. A NC-set F is said to be neutrosophic crisp closed set (NC-closed, for short) if and only if its complement F^C is a NC-open set.

Definition 1.8 [8]

Let (X, τ) be a $NCTS$ and $A = \langle A_1, A_2, A_3 \rangle$ be a NC-set in X, then A is called:
i- Neutrosophic crisp α-open set iff $A \subseteq NCint\left(NCcl\left(NCint(A)\right)\right)$.
ii- Neutrosophic crisp semi-open set iff $A \subseteq NCcl\left(NCint(A)\right)$.
iii- Neutrosophic crisp per-open set iff $A \subseteq NCint\left(NCcl(A)\right)$.

Definition 1.9 [6]

A subset A of space X is called semi-$lindelöf$ in X if any semi-open cover of A in X has a countable subcover of A.

Definition 1.10 [5]

Let (X, τ) be a $NCTS$ and $A = \langle A_1, A_2, A_3 \rangle$ be a NC-set in X, then $f: X \longrightarrow X$ is NC-semi continuous if the invers image of NC-semi open set is NC-semi open.

Definition 1.11 [14]

Let $(X, \tau\ )$ be a $NCTS$:
i- If a family $\{\langle G_{i1}, G_{i2}, G_{i3} \rangle :, i \in I\}$ of NC-semiopen sets in X satisfies the condition $X_N = \bigcup\{\langle G_{i1}, G_{i2}, G_{i3} \rangle :, i \in I\}$, then it is called a NC-semiopen cover of X.
ii- A finite subfamily of a NC-semiopen cover $\{\langle G_{i1}, G_{i2}, G_{i3} \rangle :, i = 1,2,3, \dots, n\}$ on X, is also a NC-semiopen cover of X, is called a finite subcover of NC-semiopen sets.

Definition 1.12 [8]

A family $\{\langle K_{i1}, K_{i2}, K_{i3} \rangle :, i \in I\}$ of NC-semi closed sets in X satisfies the finite intersection property (FIP for short) if every finite sub family $\{\langle K_{i1}, K_{i2}, K_{i3} \rangle :, i = 1,2,3, \dots, n\}$ of the family satisfies the condition $\bigcap\{\langle K_{i1}, K_{i2}, K_{i3} \rangle :, i = 1,2,3, \dots, n\} \neq \emptyset_N$.

Lemma 1.1 [8]

Let $U \subseteq V \subseteq X$, where X is a $NCTS$. Then U is NC-semiopen set in V, if U is NC-semiopen set in X.

Lemma 1.2 [**8**]

Let $U \subseteq V \subseteq X$, where X is a $NCTS$ and V is a NC-preopen set in X, then U is NC-semiopen (resp, NC-semi closed) in V iff $U = S \cap V$, where S is NC-semiopen (resp, NC-semi closed) in X.

Definition 1.13 [**8**]

A function from a $NCTS$ X into a $NCTS$ Y is called NC-irresolute if the inverse image of each NC-semiopen set in X, is a NC-semiopen set in Y.

Theorem 1.1 [**8**]

Let $f: (X, \tau_1) \longrightarrow (Y, \tau_2)$ be a NC-irresolute function. Then
i- if U is NC-semi-$lindelöff$ in X, then $f(U)$ is NC-semi-$lindelöff$ in Y.
ii- if U is NC-semi-compact in X, then $f(U)$ is NC-semi-compact in Y.

2- Neutrosophic Crisp Semi-Connected Spaces

Definition 2.1

A set U in $NCTS$ X is said to be NC-semi-connected set if for any two NCO-sets A & B in X such that $U \subseteq A \cup B$, then $U \cap A \neq \emptyset\ or\ U \cap B \neq \emptyset$.
In other words, U is NC-semi-connected if for any two NCO-sets A & B in X such that $A \subseteq U, B \nsubseteq U, \& A \cap B = \emptyset$.

Definition 2.2

A $NCTS$, (X, τ) is called NC-semi-connected space if for any two distinct points x & y in X, there exist a non-empty subset U in X, such that U is NC-semi-connected and $x, y \in U$.

Theorem 2.1

A $NCTS$ (X, τ), is NC-semi connected if every family of NC-semi closed sets in X having the FIP has a non-empty intersection.

Proof

Suppose that A & B are disjoint non-empty NC-semi-closed sets in X, and the family

$F = \{A^C, B^C\}$ where A^C & B^C are complement of A & B, respectively. Since A & B are disjoint, A^C & B^C are also disjoint and non-empty NC-semi open sets because A & B are NC-semi closed sets, and since every family with the FIP has a non-empty intersection, the family F also has a non-empty intersection. Then there exists an element x in X, such that $x \in A^C$ & B^C. So, this is a contradiction since A^C & B^C are disjoint. Therefore, A & B are not disjoint non-empty NC-semi closed sets in X. Hence, X is NC semi-connected.

Theorem 2.2

Let (X, τ) be a NC-semi-connected topological space. If A is an NC-open set in X, then A^C is a NC-closed set in X.

Proof

Since X, is a NC-semi-connected space, for any N-closed sets A & B in X, there exist an N-open set U in X such that $A \subseteq U$ & $B \subseteq U^C$. So that, we have $A^C \subseteq U^C$ & $B^C \subseteq U$. Then, A^C is a N-closed set in X.

Corollary 2.1

Let (X, τ) be a NC-semi-connected topological space. If A & B are a NC-sets in X such that then $A \cap B = \emptyset$, then there exists a NO-set U in X such that $A \subseteq U$ & $B \subseteq U^C$.

Proof

The proof is clear and easy.

Example 2.1

Let X be a NC-semi-connected space, and $U = \{(0,0,1), (0.1,0.3,0.6)\}$ & $V = \{(0.4,0.2,0.4)\}$ are NO-sets. Let $A = \{(0,0,1)\}$ & $B = \{(0.4,0.3,0.3)\}$. We observe that A & B are a NC-sets in X since its complement, A^C & B^C, are a NO- set.

Theorem 2.3

The finite union of NC-semi-connected sets in a $NCTS$ X, is NC-semi-connected set in X.

Proof

Let $A_1, A_2, A_3, \dots, A_n$ be NC-semi-connected sets in X, such that $A = A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$, and let B & C be N-closed sets in X such that $B \subseteq A$ & $C \subseteq A^C$, then we have $B \subseteq A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$ & $C \subseteq (A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n)^C$. Since A_i is a NC-semi-connected set, for each$i = 1,2,3, \dots, n$ there exists a N-open sets U_i in X such that $B \cap A_i \subseteq U_i$ & $C \cap A_i \subseteq (U_i)^C$. Let the NO-set $U = U_1 \cap U_2 \cap U_3 \cap \dots \cap U_n \Longrightarrow$ $\forall\ i = 1,2,3, \dots, n, \Longrightarrow (B \cap A_i) \subseteq \big(B \cap (A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n)\big) \subseteq (B \cap A) \subseteq B$, & $(C \cap A_i) \subseteq (C \cap (A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n)^C) \subseteq (C \cap A^C) \subseteq C$.
Therefore, since U is a finite intersection of NO-sets, it is also a NO-set in X. Also, we have $B \subseteq U$ & $C \subseteq U^C$. $B \subseteq U$ & $C \subseteq U^C \Longrightarrow A = A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$ is a NC-semi-connected set in X.

Corollary 2.2

Let V be NCO-set of $NCTS\ X$, and $U \subseteq V$, if U is NC-semi-connected in X, then U is NC-semi-connected in V.

Proof

Suppose that V is a neutrosophic crisp set of a $NCTS\ X$, and U is a subset of V. If U be NC-semi-connected in X, we prove U is also NC-semi-connected in V.
Since V is a neutrosophic crisp set of a $NCTS$, we can find NCO-sets N & M in X such that $A = N \cap V$, & $B = M \cap V$, where A & B are any NCO-sets in V. Since U is NC-semi-connected in X, so that for any NCO-sets N & M in X such that $U \subseteq N \cup M, U \cap N \neq \emptyset$, or $U \cap M \neq \emptyset$.
Now, suppose the NCO-sets $A = N \cap V$ & $B = M \cap V$ in V, we have $U \subseteq A \cup B$. Since $U \subseteq V$, it is clear that $U \cap A \subseteq U \cap N$, & $U \cap B \subseteq U \cap M$. Therefore, if $U \cap N \neq \emptyset$ or $U \cap M \neq \emptyset$, then $U \cap A \neq \emptyset$ or $U \cap B \neq \emptyset$. Hence, U is also NC-semi-connected in V.

Definition 2.6

A $NCTS$ is said to be semi-$lindelo\ddot{f}f$ if for every open cover of the space, there exists a countable subcover.

Definition 2.7

Let X be a $NCTS$, and let A be a subset of X. A is called a NC-semi-$lindelo\ddot{f}f$ set if and only if for every open cover $\{U_i\}$ of A, there exist a countable subset C of $\{U_i\}$ such that $A \subseteq \bigcup C$.

Corollary 2.3

Let V be NC-α-open set of $NCTS$, and $U \subseteq V$, if U is NC-semi-$lindelo\ddot{f}f$ in X, then U is NC-semi-$lindelo\ddot{f}$ in V.

Proof

Let V be a NC-α-open set of the $NCTS\ X$, $U \subseteq V$, NC-semi-$lindelo\ddot{f}f$ in X.
Suppose that C is an open neutrosophic crisp cover of U in V, and since V is a NC-α-open set, we can find open neutrosophic crisp cover $\acute{C}$ of U in X, such that each element in $\acute{C}$ is a subset of V.Since U is NC-semi-$lindelo\ddot{f}f$ in X, then there exist a countable subcover $\acute{D}$ of $\acute{C}$ that covers U, and any element in $\acute{D}$ is also a subset of V, since they are subsets of U. So that there exist a subcover D of C using the elements from $\acute{D}$, and since $\acute{D}$ is countable and any element in $\acute{D}$ is a subset of V, so D is a countable and covers U. Therefore, U is NC-semi-$lindelo\ddot{f}f$ in V.

Theorem 2.4

Let $U \subseteq V \subseteq X$, where X is a $NCTS$ and V is a NC-preopen set in X, then U is NC-semi open set in V iff $U = S \cap V$, where S is NC-semi open set in X.

Proof

To prove this, we need to show that $U \cap V^C$ is neutrosophic crisp semi open set in V.
Since $U = S \cap V$, it follows that $U \cap V^C = (S \cap V) \cap V^C = S \cap \emptyset = \emptyset \cap (V \cap V^C) = \emptyset$.
Since $U \cap V^C = \emptyset \implies U \cap V^C$ is NC-semi open in X & $U \cap V^C = \emptyset$, $U \cap V^C \subseteq V$.
Therefore, U is NC-semi open in V.

Theorem 2.5

Let V be a NC-pre-open sub set of a $NCTS\ X$, and $U \subseteq V$. If U is NC-semi-connected in X, then V is NC-semi-connected in V.

Proof

Let U be NC-semi-connected set in X, and suppose $U \subseteq V$, where V, is a NC-pre-open subset of X.
Since V is a NC-pre-open subset of X, there exist a NO-set G in X such that $G \subseteq cl(V)$, and suppose that A & B are two NO-sets in V such that $B = U \cap G^C$, & $A = U \cap G$.
Now, let $x \in A$. Since $A = U \cap G \Longrightarrow x \in U$ & $x \in G$, and since U is a NC-semi-connected set in $X, x \in A = U \cap G$. Therefore, $A \subseteq U$. Also, let $y \in B \Longrightarrow y \in U$ & $y \in G^C$.
Since $U \subseteq V$ & $G^C \subseteq \left(cl(V)\right)^C$, so $y \in U \subseteq V$ & $y \notin cl(V)$. This implies that $y \notin V$. A contradiction. Therefore, $B \nsubseteq U$.

Finally, we prove A & $B \in V, A \cap B = \emptyset$, since, $A = U \cap G$ & $B = U \cap G^C$, Hence, there exist the $\emptyset = (\emptyset) \cap A \cap B = (U \cap G) \cap (U \cap G^C) = U \cap (G \cap G^C) = U$. Thus, A & $B \in V$, satisfy $A \cap B = \emptyset$. Therefore, V is NC-semi-connected set in V.

Theorem 2.6

Let V be NC-open set of $NCTS$ and $U \subseteq V$, if U is NC-semi-connected in X, then U is NC-semi-connected in V.

Proof

Suppose that U is a NC-semi-connected subset of X, and V is a NC-open set of $X, U \subseteq V$, and let A & B be non-empty disjoint NC-open sets in V.
Since V is an open set of X, there exist an open set G in X such that $V = G \cap X$.
Note that $U = (U \cap V) = \big(U \cap (G \cap X)\big) = (U \cap G) \cap X$ Then, $U \cap G$ is an open set in X, since U is NC-semi-connected in X, $U \cap G$ cannot be divided into two non-empty disjoint NC-open sets let $M = A \cap V$ & $N = B \cap V$, which are open sets in X, so that since $V = G \cap X$, $M = \big(A \cap (G \cap X)\big) = (A \cap G) \cap X$ & $N = \big(B \cap (G \cap X)\big) = (B \cap G) \cap X$, and we can conclude that $U \cap (M \cup N) \neq \emptyset$. Since M & $N \subseteq V$, then we have $U \cap (M \cup N) = (U \cap M\ ) \cup (U \cap N) \Longrightarrow (U \cap M\ ) \neq \emptyset\ or\ (U \cap N) \neq \emptyset$, or both.
Since $M = B$& $N = A \cap V \Longrightarrow U \cap A \neq \emptyset$ & $U \cap B \neq \emptyset$. Therefor, U is NC-semi-connected in V.

Remark 2.1

The inverse of the previous theorem is not true in general case, as shown in the following example.

Example 2.2

Let $X = \{a, b, c\ \}$, and let $V \subseteq X$, defined as $V = \{a, b\}, U \subseteq V : U = \{b\}$. Then, U is NC-semi-connected in V that would separate U. However, U is not N-semi-connected in X. We can exist disjoint NO-sets in X, A & B, such that $U \subseteq A$ & $U \subseteq B$. Let $A = \{a, b\}$ & $B = \{c\}$. Then, A & B are disjoint NO-sets in X, such that $U \subseteq A$ & $U \subseteq B$.
Hence, U is not N-semi-connected in X.

Theorem 2.7

Let V be NC-α-open set of $NCTS$ and $U \subseteq V$, if U is NC-semi-$Lindel\ddot{o}ff$ in X, then U is NC-semi-$Lindel\ddot{o}ff$ in V.

Corollary 2.4

A NC-preopen subset U of NC-semi-connected X does not imply that U is NC-semi-connected in X, as shown in the following example.

Example 2.3

Let $X = \{a, b, c\ \}, U = \{a, b\}, A = \{a, b\}$ & $B = \{b, c\}$, we can observe that $U \subseteq A$ & $U \subseteq A \cup B$ & $U \cap A \neq \emptyset \neq U \cap B$, as required for U to be NC-semi-connected. However, if $U \subseteq X$, we can see $U \cap (A \cap B) U \cap \emptyset = \emptyset$, which means U is not NC-semi-connected in X.

Theorem 2.8

Let V be a NC-semi-connected set in a $NCTS\ X$ and V be is NC-semi-closed of X. Then $U \cap V$ is NC-semi-connected in X.

Proof

Since V, is a NC-semi-connected set in X, it can be expressed as $C \cup D$, where C & D are non-empty neutrosophic crisp subset such that $V \cap C \neq \emptyset \neq V \cap D$. Since U & V are NC-sets in X, then $(U \cap V)$ is also a neutrosophic crisp set in X.
Let $A = U \cap C$ & $B = U \cap D$, and since C, D, A & B are non-empty sets hence,
$(U \cap V) \cap A = (U \cap V) \cap (U \cap C) = U \cap (V \cap C) \neq \emptyset$, &
$(U \cap V) \cap B = (U \cap V) \cap (U \cap D) = U \cap (V \cap D) \neq \emptyset$.
Therefore, $(U \cap V) \cap A \neq \emptyset \neq (U \cap V) \cap B$, and $U \cap V$, can be as $A \cup B$, which means that $U \cap V$ is NC-semi-connected in X.

Corollary 2.5

A NC-semi-closed subset U of a NC-semi-connected space X is NC-semi-connected in X.

Proof

Suppose that U can not be expressed as a union of tow non-empty neutrosophic crisp subsets A & B, such that $(U \cap A) \neq \emptyset \neq (U \cap B)$. Let U be NC-semi-closed subset of X, and suppose U can be expressed as $U = A \cup B$, where A & B are non-empty neutrosophic crisp subset of X. So that, since U is NC-semi-closed, so that U^C is NC-semi-open. Then, $A \cap U^C$& $B \cap U^C$ are NC-semi-open subsets of X.
Now, if $U \cap A \neq \emptyset \neq U \cap B \Longrightarrow (U \cap A) \cap U^C \neq \emptyset \neq (U \cap B) \cap U^C$. However, for any two non-empty neutrosophic crisp semi-open subsets A & B of X, $(A \cap B) \cap (A \cup B)^C \neq \emptyset$. Hence, U cannot be expressed as $A \cup B$. This contradicts the definition of NC-semi-connected. Therefore, U is NC-semi-connected in X.

3- Functions and Neutrosophic Crisp-Semi Connected Spaces

Definition 3.1

A function $f: (X, \tau_1) \longrightarrow (Y, \tau_2)$ is NC-irresolute surjective if it is surjective and for every x in X, there exist multiple values of $f(x)$ that cover Y.

Theorem 3.1

Let a function $f: (X, \tau_1) \longrightarrow (Y, \tau_2)$ be a NC-irresolute surjective and X is NC-semi-connected, then Y is N-semi-connected.

Proof

Suppose that X is NC-semi-connected and f is a NC-irresolute and surjective function then there exist non-empty closed sets U & V in X such that $f(U) = A$ & $f(V) = B$. Since X is NC-semi-connected, there exist non-empty closed sets W in X such that $W \cap U \neq \emptyset$ & $W \cap V \neq \emptyset$. Since f is a NC-irresolute, $f(W)$ is a non-empty closed set in Y, $f(W) = A \cap B \Longrightarrow$
$f(W) \cap A\emptyset$ & $f(W) \cap B \neq \emptyset$. So that $\left(f(W) \right)$ is a non-empty closed set in Y that satisfies the the conditions required for Y to be N-semi-connected. Therefore, Y is NC- semi-connected.

Remark 3.1

Let $f: X \longrightarrow Y$ be a NC-irresolute subjective and X be NC-semi-connected, then Y be not NC-semi-connected in general as the following example.

Example 3.1

Let $X = \{a, b\}, Y = \{1,2,3\}, f: X \longrightarrow Y$ be a NC-irresolute subjective function defined as

$f(a) = 1$ & $f(b) = 2$.
It's clear that, X is NC-semi-connected, but Y is not NC-semi-connected Since we can find two disjoint NC- open sets A & B in Y, such that $A = \{1\}$ & $B = \{2,3\}$. The set $f^{-1}(A) = \{a\}$, is NC-semi-connected set in X, but A & B are disjoint in Y.

Remark 3.2

The NC-semi-continuity of f and the NC-semi-connectedness of X dose not imply the neutrosophic connectedness of Y, in general.

Example 3.2

Let $X = \{a, b, c\}$ & $Y = \{x, y\}$, such that X is NC-semi-connected set, and define $f: X \longrightarrow Y$ as follows, $f(a) = x, f(b) = y,\ f(c) = y$, it's clear that, f is a NC-irresolute surjective function. Now suppose that $Y = \{x, y\}$, then there is no direct path or connection between x & y in Y. Therefore, Y is not NC-semi-connected set.

Remark 3.3

Let X & Y be two TS_S , U be the neutrosophic set in X & $f: X \longrightarrow Y$ be NC-irresolute function. If U is NC-semi-connected in X, then $f(U)$ is not NC-semi-connected in Y in general as explain in the following example.

Example 3.3

Let $X = \{a, b, c\}, Y = \{1,2\}, f: X \longrightarrow Y \ni f(a) = f(b) = f(c) = 1, U \subseteq X, U = \{a, b,\}$, $V \subseteq Y, V = \{1\}$.
Since, U is neutrosophic semi-connected in X, the set $f^{-1}(V)$ in Y is not NC-semi-connected. Also, since $f(a) = f(b) = 1$, the set$f^{-1}(V)$ is not divided into two disjoint NC-open sets.

Theorem 3.2

Let $f: (X, \tau_1) \longrightarrow (Y, \tau_2)$ be a NCO, NC-simi-continuous function, and X be NC-semi-connected space, then Y is NC-semi-connected.

Proof

Suppose that X is a NC-semi-connected space, and f is a NC-simi-continuous function, and suppose that there exists a mapping $g: Y \longrightarrow \{0,1\}$ such that g is a continuous non-constant neutrosophic crisp.
Consider the composition $g \circ f: X \longrightarrow \{0,1\}$, it's clear that, $g \circ f$ is a continuous function from X

to $\{0,1\}$, this implies that X is not NC-semi-connected, A contradiction. Therefore, Y is a NC-semi-connected spaces.

Remark 3.4

The invers image for every NC-point $y \in Y$, is neutrosophic crisp semi-connected in X.

Theorem 3.3

Let $f: (X, \tau_1) \longrightarrow (Y, \tau_2)$ be a NC-pre-semi-closed, NC-continuous function, and $f^{-1}(y)$ is NC-semi-connected in X, for each neutrosophic point $y \in Y$. If Y is NC-semi-connected, so is X.

Proof

Suppose that Y is NC-semi-connected, and C is nonempty closed in Y, U is non-empty open in Y. Since f NC-continuous, then $f^{-1}(U)$ is open in X, also since f is a NC-pre-semi-closed, then $f^{-1}(C)$, is closed in X, also since U & C are disjoint then $f^{-1}(y)$ is NC-semi-connected are also disjoint.
Now, let y be a neutrosophic crisp point in Y. Since Y is NC -semi-connected, then $f^{-1}(y)$ is NC-semi-connected in X. Since $f^{-1}(U)$ & $f^{-1}(C)$ are disjoint, and $f^{-1}(y)$ is NC-semi-connected, so $f^{-1}(U)$ & $f^{-1}(C)$ cannot both be non-empty, then, if C is a non-empty closed set disjoint from a non-empty open set U in X, hence, C must be empty. Therefore, X is NC-semi-connected.

Theorem 3.4

Let $f: (X, \tau_1) \longrightarrow (Y, \tau_2)$ be a NC-pre-semi-closed surjective. If for each NC-point $y = \langle\{y_1\}, \{y_2\}, \{y_3\}\rangle$ in Y, $f^{-1}(y) = \langle f^{-1}\{y_1\}, f^{-1}\{y_2\}, f^{-1}\{y_3\}\rangle$ is NC-semi-connected in X, then $f^{-1}(U)$ is NC-semi-connected in X, where U is NC-semi-connected in Y.

Proof

Suppose that for each neutrosophic crisp point y in Y, the $f^{-1}(y)$ is NC-semi-connected in X. Let U be a NC-semi-connected subset of Y. hence we can express of U as $U = A \cup B$ such that A & B are non-empty closed sets in Y.
Assume that, x_1 & x_2 are arbitrary elements such that $x_1 \in f^{-1}(A), x_2 \in f^{-1}(B)$, Since f is surjective, for each element $y \in Y$, there exists element $x \in X$ such that $f(x) = y$.
Then, for each $x_1 \in f^{-1}(A)$ & $x_2 \in f^{-1}(B)$, there exist elements $y_1 \in A$ & $y_2 \in B$, such that $f(x_1) = y_1$ & $f(x_2) = y_2$.
Since, A & B are non-empty closed sets in Y, and $f^{-1}(y_1)$ &$f^{-1}(y_2)$ are NC-semi-connected in X, there exist closed sets C_1 & C_2 in X such that
$C_1 \cap f^{-1}(y_1) \neq \emptyset$ & $C_2 \cap f^{-1}(y_2) \neq \emptyset$, let $C = C_1 \cap C_2 \Longrightarrow C = f^{-1}(A) \cap f^{-1}(B)$. Then C is a closed non-empty set in X.
Hence, $f^{-1}(U) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, $C \subseteq f^{-1}(U)$. Therefore, $f^{-1}(U)$ is NC-semi-connected in X.

Remark 3.5

In general case, if $f: X \longrightarrow Y$ is NC-semi-connected in X, for each NC-point $y = \langle\{y_1\}, \{y_2\}, \{y_3\}\rangle$ in Y. If Y is NC-semi-connected, so X is not NC-semi-connected as explain in next example.

Example 3.4

Let $f: X \longrightarrow Y$, where $X = \{a, b\}$ & $Y = \{1,2,3\}$. Let $f(a) = 1,\ f(b) = 0$. Now, let

$f^{-1}(y) = 1$. The $f^{-1}(1) = \{a\}$, which is a NC-semi-connected set in X.
If we consider $f^{-1}(y) = 1$, the $f^{-1}(2) = \{a\}$, which is not NC-semi-connected set in X.
Therefore, X is not NC-semi-connected.

Conclusion

The paper deals with the concept of semi-connectedness (resp. semi-$lindelöf$) in the generalized setting of a neutrosophic crisp topological space. We achieve a number of a neutrosophic crisp semi-connected (resp. neutrosophic crisp semi-$lindelöf$) space. Also, we introduce and study the concept of neutrosophic crisp locally semi-connected spaces, neutrosophic crisp super-connected spaces, neutrosophic crisp-strongly connected spaces, and study their properties.

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