

## Assessment and Performance Comparison between Patch and Rectangular Resonators through Suitable Computational Method

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### Abstract

This study explores the analysis of lossy microwave structures, focusing particularly on cavity and microstrip resonators, in terms of characteristic impedance, Q factor, and the Transmission Line Matrix (TLM) method. Microwave structures are pivotal in various applications, from consumer electronics to communication systems, due to their ability to manipulate microwave frequency signals. The research delves into the computation of conductor and dielectric losses, which are essential for understanding microwave circuit performance. Utilizing the Finite Difference Time Domain (FDTD) and TLM methods, the study compares the Q factor and visualizes the electric field distribution (Ez field) in both microstrip and cavity resonators. This comparison highlights the distinct energy storage and reflection characteristics inherent to each resonator type. Through simulation work and analysis, the paper elucidates the complex interplay between the physical design parameters and the operational efficacy of these resonators, thereby offering insights into optimizing microwave systems for enhanced performance.

**Index Terms – TLM , FDTD ,TEM, Q Factor.**

### I. INTRODUCTION

Microwave structures are physical configurations designed to guide, control, and manipulate microwave frequency signals. Electromagnetic waves, or microwaves, have frequencies between around 300 MHz and 300 GHz. In many different applications, including microwave ovens, radar systems, and satellite communications, the structures operating at these frequencies are essential. Waveguides, coaxial cables, microstrip lines, strip lines, and cavity resonators are a few typical microwave structures.

Microwave components tend to include distributed components, meaning that their dimensions are on the order of the electrical wavelength, and the phase of the voltage or current varies dramatically throughout the physical extent of the device. At substantially lower frequencies, the wavelength is sufficiently large to provide minimal phase fluctuation throughout a component's dimensions. Optical engineering, where the wavelength is substantially shorter than the component's size, is the other extreme of frequency. In this instance, the geometrical optics regime may be used to simplify Maxwell's equations and construct optical systems using geometrical optics theory. [1].

Resonators and other lossy structures are essential to the area of microwave engineering for a variety of uses, such as frequency control and filtering. An investigation of characteristic parameters for a lossy resonator is one example of contemporary academic work in this field. The calculation of the electromagnetic field distribution in a resonant cavity with different loss dielectrics is explored in this study.

Microwave materials can be categorized according to their distinct characteristics as well as their chemical or physical structure. Complex permeability and complex permittivity are the main

factors influencing a material's characteristics at microwave frequencies. The choice of a suitable material for a given application is mostly determined by the material's intrinsic losses as well as its electrical and/or magnetic characteristics. Generally expressed as loss tangents, the dielectric and magnetic losses depend on the operating frequency [2].

Several composites, such as low-loss dielectric ceramics, low-loss polymer ceramic composites, dielectric resonators, and multilayer ceramics, are researched and produced for certain applications since it is challenging to discover a single material that possesses all these qualities. Numerous research publications have been published in this sector after these materials were thoroughly investigated for a variety of microwave applications [3].

The majority of microwave applications also require the use of an appropriate computational method; the most complicated and potent method utilized in this sector is the Transmission Line Matrix method (TLM). TLM is published in 1971 where is created by Johns and Beurle and initially has become a crucial numerical technique in computational electromagnetics [4]. Originally TLM is based on the analogy between the electromagnetic field and a mesh of transmission lines [5]. The TLM method allows to model complex electromagnetic structures [6]. As a network model of Maxwell's equations formulated in terms of the scattering of impulses, it possesses exceptional versatility, numerical stability, robustness, and isotropic wave properties.

In this research we consider some analysis on a specific type of lossy Microwave structure (which will be a cavity resonator) in contrast of microstrip resonator in prospective of characteristic impedance and Q factored.

## II. LOSSY MICROWAVE SYSTEMS

Lossy microwave structures are an essential aspect of microwave engineering, where energy dissipation (or loss) is considered within microwave transmission lines and resonant structures. These losses are crucial for understanding the performance of microwave circuits, including filters, amplifiers, and antennas. The losses in microwave structures can be attributed to various factors, including dielectric losses, conductor losses, and radiation losses. Here, some key concepts and equations related to lossy microwave structures.

### 1- Conductor Losses.

Conductor losses arise due to the finite conductivity of the materials used to construct the microwave transmission lines or resonators. The skin effect, where alternating current tends to flow near the surface of the conductor at high frequencies, also influences these losses. The conductor loss per unit length ( $\alpha_c$ ) can be approximated by:

$$\alpha_c = \frac{R_s}{2Z_0} \quad (1)$$

Where  $R_s$  is the surface resistance of the conductor, and  $Z_0$  is the characteristic impedance of the transmission line.

### 2- Dielectric loss.

Dielectric losses result from the energy dissipated as heat when the electromagnetic field interacts with the dielectric material within the microwave structure. These losses are characterized by the dielectric loss tangent ( $\tan\delta$ ).

The dielectric loss per unit length ( $\alpha_d$ ) can be expressed as:

$$\alpha_d = \frac{\pi f \epsilon''}{c} = \frac{\pi f \epsilon' \tan \delta}{c} \quad (2)$$

where  $f$  is the frequency of operation,  $\epsilon''$  is the imaginary part of the dielectric constant,  $\epsilon'$  is the real part of the dielectric constant,  $\tan \delta$  is the loss tangent, and  $c$  is the speed of light in vacuum.

### 3- Quality Factor (Q).

The quality factor (Q) of a resonant structure is a measure of its resonator quality, indicating how underdamped the resonator is. For lossy resonators, Q is inversely proportional to the total losses:

$$Q = \frac{\omega_0 W}{P_{loss}} \quad (3)$$

#### A. Microstrip line structure.

A dielectric substrate with a strip conductor on one side and a ground plane on the other makes up a microstrip line, as shown in Figure 1. The microstrip, in contrast to the strapline, is essentially an open structure that needs substrates with a high dielectric constant in order to restrict electromagnetic fields close to the strip conductor. In addition, the microstrip line is a non-uniform structure. True TEM propagation, or the existence of a pure TEM mode, is impossible due to the composite nature of the dielectric contact. At the dielectric surface, the boundary criteria for this mode cannot be satisfied.

On the other hand, the mode of propagation at low frequencies is called the quasi-TEM mode because it closely resembles the TEM mode. The dielectric substrate underneath the strip conductor is where the electric and magnetic field lines are mostly concentrated, and the air region above has slightly less of them [7].

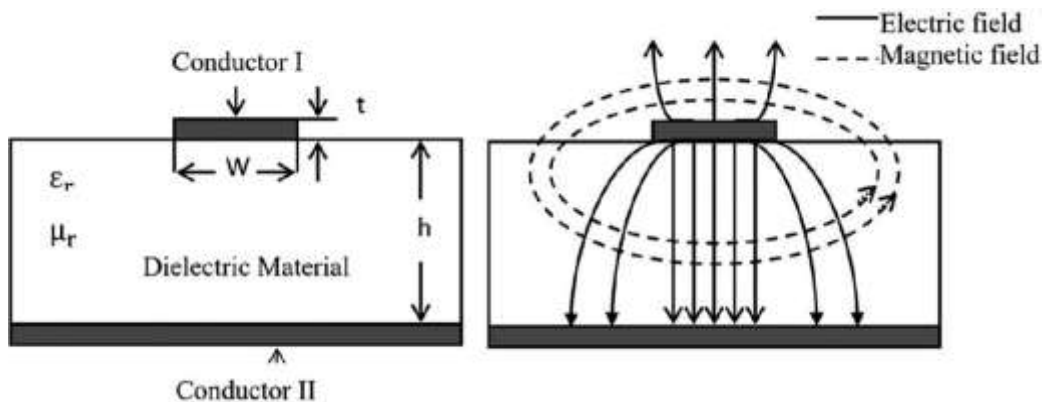


Fig. 1. Cross-sectional view and field configurations of microstrip

The concentration of energy in the substrate region will increase as the substrate's relative dielectric constant ( $\epsilon_r$ ) increases. Due to its open design, it is simple to put discrete devices in series with it and to make small adjustments once the circuit has been constructed. But precautions need to be taken to reduce radiation loss or interference from adjacent wires.

Because they would result in smaller circuit dimensions and a lower phase velocity, using substrates with a high dielectric constant could be useful. Given that the structure transforms into a mixed dielectric transmission line, the microstrip analysis becomes a little more difficult. Up to

35 GHz, the microstrip is a flexible transmission line for millimeter-wave and microwave integrated circuits. By utilizing thin dielectric constant substrates and insulating the structure, its operating frequency range can be increased to 94 GHz [8].

*B. Applications of Microstrip line.*

Microwave systems, including measurement instruments where low-loss and high-power characteristics are not strictly required, make extensive use of microstrip line-based filters, impedance transformers, hybrids, couplers, power dividers and combiners, delay lines, baluns, circulators, and antennas. The microstrip section can be employed as a lumped element if its size is lowered to significantly smaller dimensions than the wavelength. In both passive and active hybrid and monolithic integrated circuits, microstrip sections are frequently utilized in lumped and distributed configurations.

*i. Characteristic Impedance and effective dielectric constant.*

There are two main parameters need to be considered in analyses of microstrip line include The Characteristic Impedance and effective dielectric constant, Because there are several layers of dielectric materials involved, it may be more difficult to compute the characteristic impedance of a microstrip line in an isotropic multilayer dielectric environment. The permittivity of each layer could vary, affecting the electromagnetic fields and, consequently, the microstrip line's impedance.

The conductive strip's dimensions, the thickness of the dielectrics, and the dielectric materials' dielectric constant ( $\epsilon_r$ ) determine the characteristic impedance ( $Z_0$ ) of a microstrip line. It is necessary to compute the effective dielectric constant ( $\epsilon_{eff}$ ) for an isotropic multilayer dielectric by considering the contributions of each layer. The general method is to solve the structure's Maxwell's equations while considering the boundary conditions at each dielectric contact.

A simplified equation that is often used for a single homogeneous, isotropic dielectric layer is given by:

$$Z_0 = \frac{z_{0air}}{2\pi\sqrt{2(\epsilon_r+1)}} \ln\left(\frac{8h}{w} + \frac{w}{4h}\right) \quad (5)$$

Where  $z_{0air}$  is the impedance of free space (approximately 377 Ohms),  $h$  is the height of the dielectric substrate,  $w$  is the width of the microstrip, and  $\epsilon_r$  is the relative permittivity of the dielectric material.

To account for the effects of each layer,  $Z_0$  for multilayer dielectrics must be calculated using more complex models or by changing the above method. Determining the effective permittivity ( $\epsilon_{eff}$ ) would include calculating the relative thickness and permittivity of each layer, resulting in some sort of average [9].

For a microstrip line, the characteristic impedance  $Z_0$  in terms of the effective dielectric constant  $\epsilon_{eff}$  can be represented by different formulas depending on the width-to-height ratio of the microstrip ( $\frac{h}{w}$ ). Two common formulas used for calculating  $Z_0$  based on  $\epsilon_{eff}$  are:

- 1- For a microstrip line where the width  $w$  is less than or equal to the height  $h$  of the dielectric substrate ( $\frac{w}{h} \leq 1$ ):

$$Z_0 = \frac{60}{\sqrt{\epsilon_{eff}}} \ln \left( \frac{8h}{w} + \frac{w}{4h} \right) \quad (6)$$

2- For a microstrip line where the width  $w$  is greater than the height  $h$  of the dielectric substrate ( $\frac{w}{h} > 1$ )

$$Z_0 = \frac{120\pi}{\sqrt{\epsilon_{eff}} \left( \frac{w}{h} + 1.393 + 0.667 \ln \left( \frac{w}{h} + 1.444 \right) \right)} \quad (7)$$

And the  $\epsilon_{eff}$  itself for a microstrip line depends on the equivalent relative permittivity of the substrate layers ( $\epsilon_{reqi}$ ) and the geometry of the microstrip. For a homogeneous isotropic dielectric layer, a commonly used formula for  $\epsilon_{eff}$  is:

$$\epsilon_{eff} = \frac{\epsilon_{reqi} + 1}{2} + \frac{\epsilon_{reqi} - 1}{2} \left( \frac{1}{\sqrt{1 + 12 \frac{w}{h}}} \right) \quad (8)$$

### ii. Microstrip Resonator.

In basic terms, a microstrip resonator is a tiny transmission line segment with the ability to store energy in the form of electromagnetic oscillations at specific frequencies. A quarter of the wavelength  $\lambda$  of the resonant frequency  $f_0$  is represented by the line length  $L$  in the simplest type of resonator, known as a quarter-wave resonator. The formula defines the resonant frequency is:

$$f_0 = \frac{v_p}{4L} \quad (9)$$

where,  $v_p$  represents the phase velocity of the signal in the microstrip, which is related to the speed of light  $c$ , and the effective dielectric constant  $\epsilon_{eff}$  of the substrate material is given by:

$$v_p = \frac{c}{\sqrt{\epsilon_{eff}}} \quad (10)$$

Also, for rectangular microstrip patch antenna the resonance frequency for any  $TM_{mn}$  mode is given by [10]:

$$f_0 = \frac{c}{2\sqrt{\epsilon_{eff}}} \left[ \left( \frac{m}{L} \right)^2 + \left( \frac{n}{W} \right)^2 \right]^{\frac{1}{2}} \quad (11)$$

where using the LC model the resonance frequency could represent by :

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad (12)$$

Where the C is the capacitor and L is the Inductor. So the quality factor will be as follow:

$$Q = \frac{2\pi f_0 c}{G_0} \quad (13)$$

Where is the  $G_0$  is the conductance, which represents the energy dissipated in the resonator.

### iii. Rectangular Cavity resonator.

An essential part of optical and microwave integrated circuits are cavity resonators. A cavity resonator is a relatively basic device consisting of a segment of hollow metal waveguide that is shorted at both ends and has dimensions similar to the wavelength. As seen in Fig. 2, the hollow is often rectangular or cylindrical. The cavity resonator is made from a rectangular wave guide portion that has two more metal plates on it to shut it off.

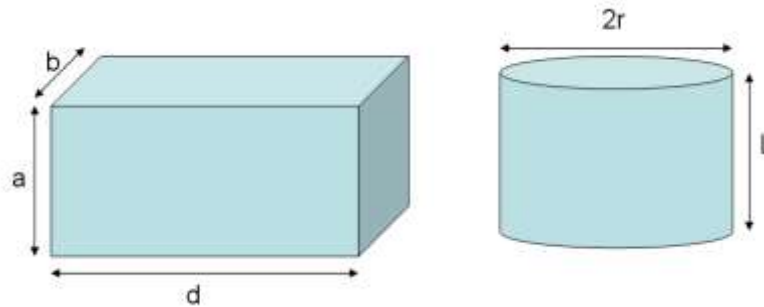


Fig. 2. Basic geometries of (left) a rectangular cavity and (right) a cylindrical cavity.

The resonance can offer a secondary coupling link between the energy source and the target at the cavity resonance frequencies. Furthermore, the resonance's field structure may significantly affect the cavity's circuit impedances, with uncertain implications for circuit performance. The ratio of wavelength to cavity size may be used to compute the resonant frequencies of a cavity. These frequencies are contingent upon the cavity's size and the electromagnetic fields' boundary conditions. The resonant frequencies for a rectangular cavity may be obtained using Maxwell's equations, and they are provided by:

$$F_{mnq} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{q}{d}\right)^2} \quad (14)$$

Where is  $F_{mnq}$  is the resonant frequency for the  $m, n, q$  mode,  $c$  is the speed of light, and  $a, b,$  and  $d$  are the dimensions of the cavity.  $m, n,$  and  $q$  are the mode numbers in the  $x, y,$  and  $z$  directions, respectively. This equation assumes that the cavity is filled with a non-conductive medium (like air).

#### IV. NUMERICAL METHODS

Numerical methods of solving Maxwell's equations have gained immense popularity with the development of digital computers. Because of the great accuracy with which Maxwell's equations capture electromagnetic physics in the natural world, numerical solutions derived from solving his equations are frequently more trustworthy than experimental results. Another name for this area of study is computational electromagnetics. Computational electromagnetics consists mainly of two kinds of numerical solvers: one that solves the differential equations directly, the differential-equation solvers; and one that solves the integral equations which are derived from Maxwell's equations.

In this paper will concentrate on two main types of numerical methods that considered useful in the electromagnetic problems, transmission line matrix method (TLM) and finite difference time domain (FDTD), those methods can solve complicated problems, but it is generally computationally expensive where is the solutions may require a large amount of memory and computation time.

##### A. Transmission Line Matrix Method.

The discrete representation of the electromagnetic phenomena in space and time is the Transmission Line Matrix (TLM) technique. The first presentation of the TLM approach was in 1974 by Mr Johns , as a two-dimensional method [11], and it was expanded into three dimensions

in 1987 with the creation of the symmetrical condensed node [12]. Since then, several attempts have been made to show the method's validity by directly deriving it from Maxwell's equations [13].

This approach's main assumption is that, when seen from a wider perspective, the TLM method may be thought of as a cellular automaton (CA). The first person to investigate the concept of cellular automata was John von Neumann [14]. Independent computing units make up cellular automata. These units exchange values with nearby units to modify their states. The nature of a cellular automaton is inherently parallel.

The state of a TLM cell can be described by a vector of wave quantities. The transformation from one state to another (the computation of one cell) is represented by a scattering process. So, if the information about the state before scattering is stored in a vector and the result of the scattering (computation) in a vector  $b$ , we may express the computation process of a TLM cell as:

$$b = Sa \quad (15)$$

where  $S$  is the scattering matrix. The scattering matrix defines the computation performed by a cell.

The two-dimensional TLM method is suitable for the analysis of electromagnetic fields with the electric field components oriented normal and the magnetic field parallel to a certain plane of reference (TE case), or vice versa the magnetic field components oriented normal and the electric field parallel to the plane of reference (TM case) [16]. Fig. 3a shows a TE arrangement with two parallel conducting plates. This arrangement may be modelled by a two-dimensional mesh of lines as depicted in Fig. 3b.

A standard 2D TLM shunt node is shown in Fig. 5. The characteristic impedance of the link lines is  $Z_0 = \sqrt{L/C}$ . The interconnection at the center of the node will be called cell center. The cell center is delay-free, frequency independent and energy conservative. The scattering matrix of a shunt cell center equals to the scattering matrix of a parallel adaptor of wave digital filter (WDF) [15].

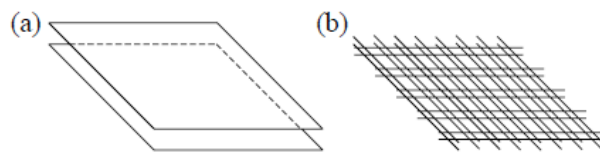


Fig.3. (a) Parallel plate, (b) 2-D mesh.

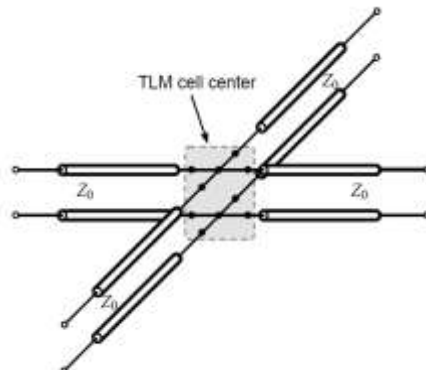




Fig. 4 . 2D TLM shunt node and its TLM cell center.

Let us assume a quadratic mesh with a spatial separation  $\Delta l$ . The propagation delay  $\Delta t$  of a voltage pulse scattered in a node is given by:

$$\Delta t = \frac{\Delta l}{cm} \quad (16)$$

Where  $cm$  is wave velocity on the mesh lines .

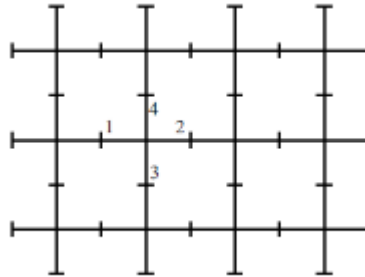


Fig. 5 . 2-D TLM mesh.

### B. Finite Difference Time Domain Method.

The finite-difference time-domain (FDTD) approach is perhaps the most straightforward full-wave methodology for solving electromagnetics issues. It can correctly solve many kinds of issues. Like any numerical approaches, though, it has its share of artifacts, and the accuracy depends on how it is put into practice.

Although the FDTD approach is often computationally costly, it may deal with challenging issues. A significant amount of memory and processing time may be needed for the solutions. The FDTD method is roughly classified as a "resonance region" approach, meaning that its characteristic dimensions are roughly equivalent to a wavelength in size [17].

More efficient solutions are usually obtained with quasi-static approximations when the object is relatively tiny in relation to a wavelength. Alternatively, different techniques, such as ray-based methods, may offer a far more effective way to tackle the problem if the wavelength is extremely tiny in comparison to the physical properties of interest.

In the FDTD simulation to get best results must follow the following steps: -

- An FDTD mesh (or grid) must be created for the problem.
- This mesh must be fine enough where  $\Delta s$  must be no more than 1/10 of the minimum wavelength (i.e. maximum frequency) of interest (which  $\Delta s$  represent the spatial step size) .
- The time step  $\Delta t$  must satisfy the Courant condition.

$$\Delta t = \frac{\Delta x}{\sqrt{2} c} \quad (16)$$

where  $C$  is speed of light,  $\Delta x$  is the spatial offset.

- An appropriate signal shape (e.g. differentiated Gaussian) with suitable time duration for the desired spectral content must be chosen.

Now with apply the following FDTD equations:



$$H_x(i, j) = H_x(i, j) - \frac{\Delta t}{\mu_0 * \Delta y} (E_z(i, j + 1) - E_z(i, j)) \quad (17)$$

$$H_y(i, j) = H_y(i, j) - \frac{\Delta t}{\mu_0 * \Delta x} (E_z(i, j + 1) - E_z(i, j)) \quad (18)$$

$$E_z(i, j) = A E_z(i, j) + B (H_y(i, j) - H_y(i - 1, j) - H_x(i, j) + H_x(i, j - 1)) \quad (19)$$

Where A and B precompute coefficients as scalars calculated by the following equations:

$$A = \frac{\left(1 - \frac{\sigma \Delta t}{2\epsilon_0 \epsilon_r}\right)}{\left(1 + \frac{\sigma \Delta t}{2\epsilon_0 \epsilon_r}\right)} \quad (20)$$

$$B = \frac{\left(\frac{\Delta t}{\epsilon_0 \epsilon_r \Delta x}\right)}{\left(1 + \frac{\sigma \Delta t}{2\epsilon_0 \epsilon_r}\right)} \quad (21)$$

Where  $\sigma$  is the conductivity.

## V. SIMULATION WORK AND ANALYSES.

To calculate the start energy, losses, Q factor for the microstrip and cavity resonator we applied the following simulation will the specific parameters value as it illustrated in the table [1].

Table 1. simulation parameters

Parameter	Microstrip	Cavity
C	$3 \times 10^8$	$3 \times 10^8$
$\epsilon_r$	2.5	2.25
$\sigma$	$5.7 \times 10^7$	$5.7 \times 10^7$
Dimensions	(a=5mm, b=4mm, d=7mm)	(a=6 mm, b=5mm, d=8mm)
Frequency range	From 1GHz to 30 GHz	From 1GHz to 30 GHz

By using TLM method as shown in Fig 6. The resonance frequency occurred at 23 GHz in microstrip resonator for the stored energy while the resonance for the storage energy was on the while in cavity resonator was all frequency except the range from 7 to 15 GHz.

By using FDTD Method to calculate the Q Factor, which is get the following results that shows the superiority of the microstrip resonator over the cavity resonator as in Fig. 8 , the resonance frequency of the Cavity is 21 GHz ,note this result comes from 200 iteration , while there is enhancement with apply 1000 iteration as in Fig . 9.

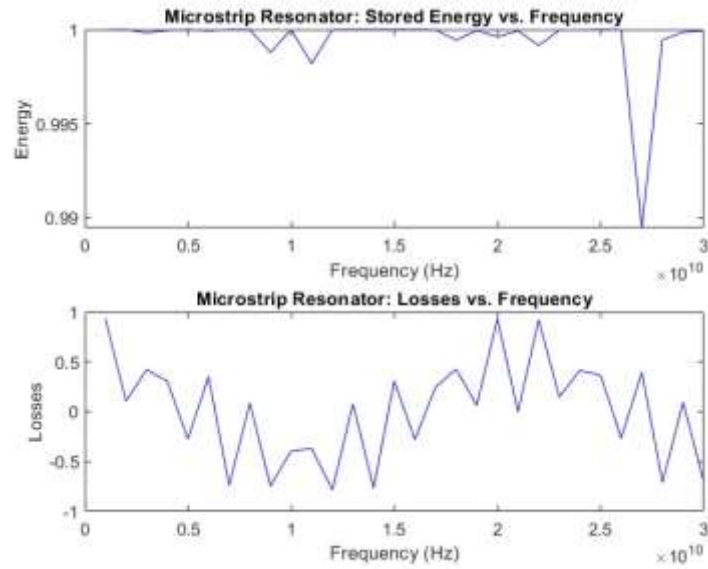


Fig 6. Microstrip resonator results with TLM method.

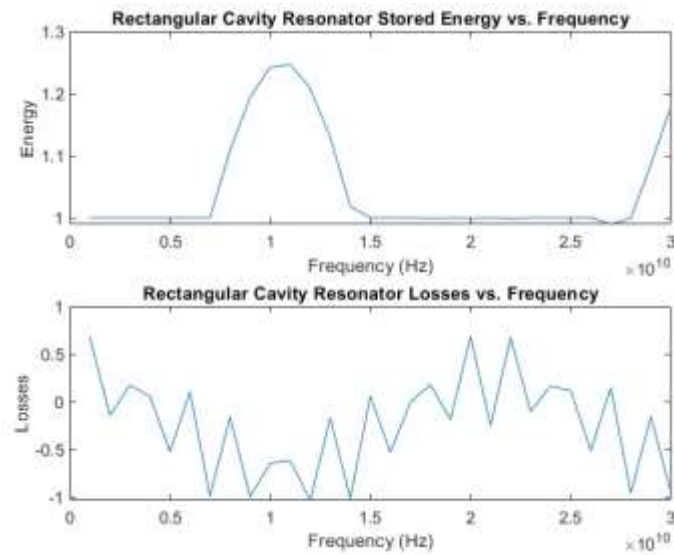


Fig 7. Cavity resonator results.

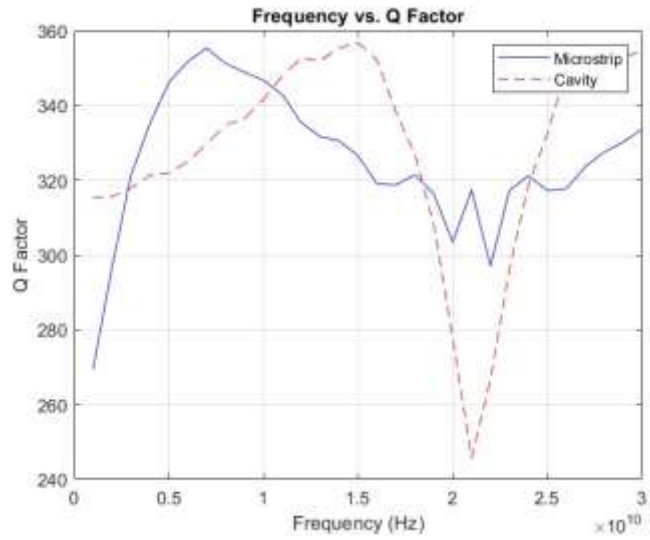


Fig 8. Q factor results with 200 iteration of FDTD.

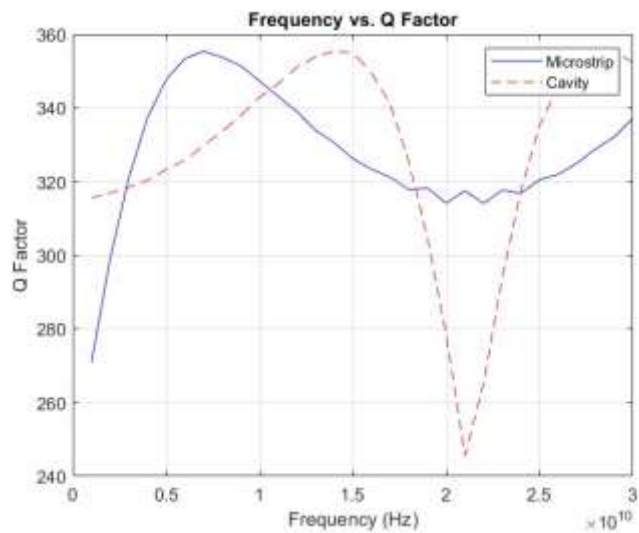


Fig 9. Q factor results with 1000 iteration of FDTD.

Also with using FDTD method to visualize the Ez field for the microstrip and cavity resonator as shows in Fig.10 and Fig.11 , where it is clear the reflection from the wall of the cavity resonator while there is no reflection in microstrip resonator.

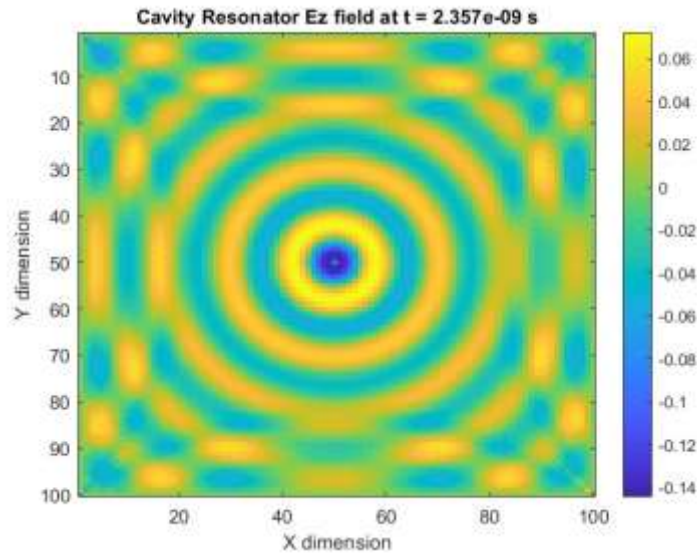


Fig 10. Ez field in Microstrip Resonator

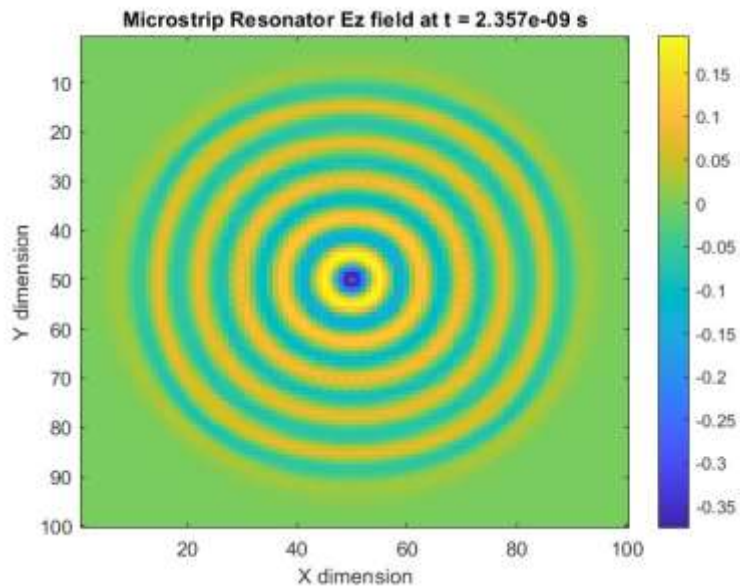


Fig 11. Ez field in Rectangular Resonator.

## VI CONCLUSION .

The comparative analysis of lossy microwave structures, specifically focusing on microstrip and cavity resonators, has yielded significant insights into their operational characteristics and performance metrics. Through the application of the FDTD method and the TLM technique, this study has successfully quantified the Q factor and visualized the  $Ez$  field distribution for both resonators. The findings demonstrate that microstrip resonators exhibit superior Q factor performance and lack wall reflections, unlike their cavity counterparts, which show distinct energy reflections. These results underline the importance of considering both physical design constraints and electromagnetic properties when designing microwave systems.

Ultimately, this research contributes to the broader field of microwave engineering by providing a nuanced understanding of how lossy structures behave under varying conditions, thereby guiding the development of more efficient and effective microwave components and systems.

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