Thermal Analysis of Vertical U-tube Ground Source Heat Exchanger (VGSHE) and Optimization of its Performance

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Abstract

The purpose of this paper is to optimize the length and the flow rate of the vertical U-tube heat exchanger (VGSHE) by minimizing the entropy generation rate. The estimation of vertical U-tube heat exchanger performance is not easy as of the complicated heat transfer conditions of its design. Therefore, the analysis is based on the following assumptions " The ground (soil) temperature is not a function of neither depth nor time, and the wall temperature of U-tube is constant along the length of the borehole and is equal to the ground temperature ($T_w = T_g$) and the water physical properties do not change with temperature". As a result, the fluid temperature increases until it reaches the maximum temperature and that means there is a length where the fluid gets its maximum temperature. This length is the optimal length of the system.

Keywords: Renewable energy; VGSHE; U-tube heat exchanger; entropy generation; heat pump; thermal heat transfer; optimization

الملخص

تهدف هذه الورقة لإيجاد الطول و معدل التدفق الامثل للمبادل الحراري الراسي على شكل حرف *(U)* *(VGSHE)* عن طريق تقليل معدل توليد الانتروبي. ان تقدير اداء المبادل الحراري الراسي علي شكل حرف *(U)* ليس سهلا نظرا لظروف نقل الحرارة المعقدة لتصميمه. لذلك يعتمد التحليل علي الافتراضات التالية "درجة حرارة الارض (التربة) ليست دالة في العمق و لا في الزمن و درجة حرارة جدار الانبوب *(U)* ثابتة على طول الثقب و تساوي درجة حرارة الارض ($T_w = T_g$) و ان الخواص الفيزيائية للماء لا تتغير مع درجة الحرارة " و نتيجة لذلك ترتفع درجة حرارة السائل (الماء) حتى تصل الى اعلى درجة حرارة و هذا يعني ان هنالك طول للأنبوب يصل فيه السائل الى درجة الحرارة القصوى. هذا الطول هو الطول الامثل للنظام

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1. Introduction

The ground is one of the free sources of energy that can be exploited to provide any system with clean and cheap energy. Consequently, the ground coupled heat pump (GCHP) has been given considerable interest as it is one of the best renewable energy technologies recently. It has proved its efficiency of providing heating and cooling for residential and commercial buildings since the temperature of the ground is relatively constant. A typical GCHP system consists of a conventional heat pump coupled with a ground heat exchanger. The principle of heat pump operation is not different from refrigeration equipment. There are two types of ground coupled heat pumps:

- Horizontal Ground Source Heat Exchanger (HGSHE)
- Vertical Ground Source Heat Exchanger (VGSHE)

Vertical ground coupled heat pump as shown in Figure (1) is commonly used since it does not need more space for installation than horizontal ground coupled heat pump does. On the other hand, it is more expensive for installation than horizontal one. The diameter of the tube is usually between 1 to 2 inches.

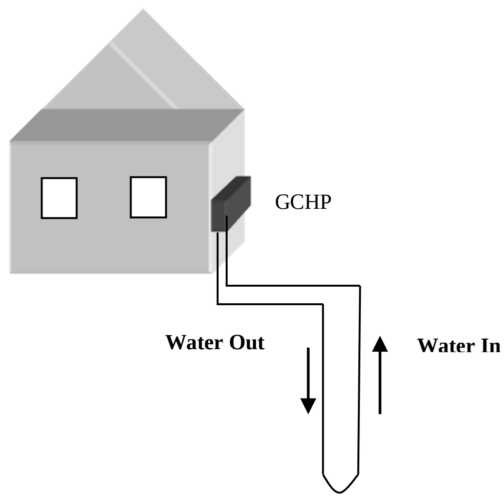

Fig. 1 Vertical Ground Heat Pump

In lately years, many researches have been done for simulating this system. Most of these researches used the numerical methods for simulation the heat transfer operation of the system such as Rottmayer [1] in 1997. In 2009, Elmozughi [2] used Gambit and Fluent software to simulate the system. In this paper, the analytical solution is used to obtain the distribution temperature equations along the U-tube and then using the second law of thermodynamic to optimize the length of the U-tube by minimizing the entropy generation, Table (1) shows the Constant Parameters Abbreviations.

Table(1) Constants Parameters Abbreviations

A	constant parameter (m^{-1})	Greek symbols:	
A_s	*surface area* (m^2)	P	*density* ($kg\ m^{-3}$)
B	parameter (m^{-1})	μ	*vescosity* (pas)
C_p	*constant pressure specific heat* ($J\ kg^{-1}\ K^{-1}$)	Λ	*eigenvalues* (m^{-1})
		Ω	*parameter* (m^{-1})
K_{gr}	*thermal conductivity of the grout* ($W\ m^{-1}\ K^{-1}$)	Β	*parameter* (K)
		θ_i	*intial temperature difference between fluid and wall* $(T_i - T_g), (C)$
H	*heat transfer coefficient* ($W\ m^{-2}\ K^{-1}$) & *enthalpy* ($J\ kg^{-1}$)	θ_1	*temperature difference between fluid and wall at the inlet branch* $(T_1 - T_g), (C)$
H	*depth of the borehole* (m)		
L	*the length of the U − tube* (m)		
$\dot{m}$	*mass flow rate* ($kg\ sec^{-1}$)	θ_2	*temperature difference between fluid and wall at the outlet branch* $(T_2 - T_g), (C)$
r_t	*raduis of the U − tube* (m)		
Re	*Reynolds Number*		

S	$shape\ factor\ (m)$	Φ	$parameter\ (W\ m^{-1}\ K^{-1})$
$\dot{S}_{gen}$	$entropy\ generation\ (W\ K^{-1})$	δ	$parameter\ (W\ m^{-1}\ K^{-1})$
W	$centered\ distance\ between\ the\ two\ branches\ of\ U-tube\ (m)$	V	$eigenvectors$
		N	$specific\ volume\ (m^3\ kg^{-1})$
T_1	$temperature\ of\ the\ inlet\ branch\ along\ x\ direction\ (C)$	ξ_1, ξ_2	$non-dimensional\ parameters$
		Ƒ	$friction\ factor$
T_2	$temperature\ of\ the\ outlet\ branch\ along\ x\ direction\ (C)$	E	$roughness\ of\ the\ pipe$
		Subscripts :	
T_g	$temperature\ of\ the\ ground\ (C)$	B	Branch
q'	$total\ heat\ transfer\ rate\ per\ unit\ length\ (W\ m^{-1})$	G	Ground
		g_r	Grout
$\acute{q}_g$	$heat\ convection\ flux\ per\ unit\ length\ due\ to\ heat\ transfer\ from\ the\ ground\ (W\ m^{-1})$	H.Ex.	Heat Exchanger
		I	Inlet
		P	$Pressure\ (N\ m^{-2})$
$\acute{q}_b$	$heat\ conduction\ flux\ per\ unit\ length\ due\ to\ temperature\ difference\ between\ the\ two\ branches\ of\ the\ U-tube\ (W\ m^{-1})$	P_r	$Prandlt\ number$
		S	Section
		T	Tube
Qmax	*Total heat load form heat exchanger (Watt)*		

2- Mathematical model of the system

2. 1. Analytical Solution

The U-tube heat exchanger Figure (2) is buried vertically in the borehole in the ground. The borehole is filled with grout which can be a variety of materials ranging from concrete to sand. The working fluid that is used for exchanging heat with refrigerant can be pure water, a mixture of water and anti-freeze, glycol solution, or brine. The fluid exchanges the heat with the ground as it goes down along the U-tube and returns to the heat pump. The branches of the U-tube, also called legs of the U-tube, will be referred as cold side (1) and hot side (2).

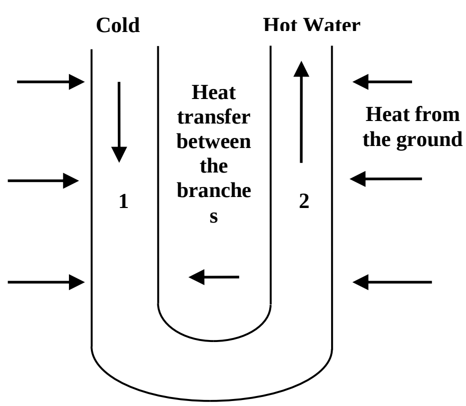

Fig.2 Vertical U-tube Heat Exchanger Model

As the above figure (2) shows a unique heat transfer situation is represented from this U-tube geometry. The ground heats the U-tube, beside that; there is heat exchange between the legs. As the time passes from the starting operation time, the branch (2) starts to loss heat to the branch (1). Thus this thermal interference reduces the amount of the energy that is gained from the ground. This thermal interference is also called thermal coupling. Since the grout separates these two branches, this amount of the energy that is transferred from branch (2) to branch (1) depends on this conductivity of the grout. Accordingly, the gained energy from the ground decreases when the amount of the heat transfer between the legs increases. Figure (3) illustrates these two heat fluxes that are absorbed by working fluid. These two heat fluxes are from the ground (dq_g) and between the two branches (dq_b).

The heat flux from the ground is assumed to be absorbed radially at steady state heat conduction. The area around the heat exchanger can be divided into three coaxial cylinders which are U-tube, grout and soil. The heat transfer coefficient between can be found from Holman [3].

$$h = \frac{1}{\sum_{i=1}^{3} \frac{r_1}{k_i} \ln \frac{r_i}{r_{i-1}}}$$

Where k is the conductivity coefficient, and r1, r2 & r3 are the radiuses of the each coaxial cylinders of the pipe, grout and where the ground temperature is constant in the soil, respectively.

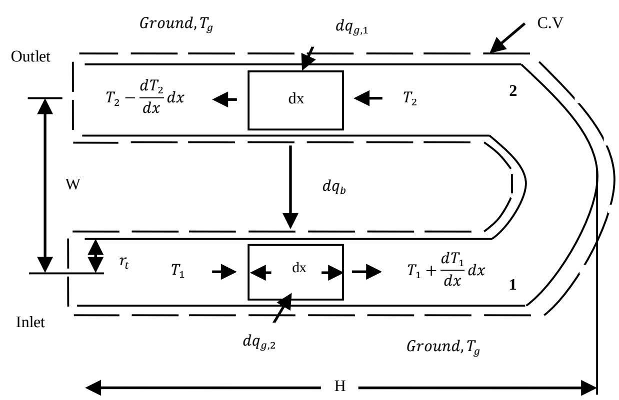

Fig. 3 Heat balance of the U-tube heat exchanger

$$-\dot{m}C_p \frac{dT_1}{dx} dx + dq_{g,1} + dq_b = 0 \text{......(1)}$$

$$\dot{m}C_p \frac{dT_2}{dx} dx + dq_{g,2} - dq_b = 0 \text{......(2)}$$

Where

$$dq_{g,1} = hA_s (T_g - T_1)$$

$$dq_{g,2} = hA_s (T_g - T_2)$$

$$dq_b = Sk_{gr} (T_2 - T_1)$$

$$A_s = 2\pi r_t dx$$

$$S = \frac{2\pi H}{\cosh^{-1}\left[\frac{W^2}{2r_t^2} - 1\right]}$$

Where A_s is the cross-sectional area of U-tube heat exchanger. The U-tube heat exchanger can be modeled as two very long identical cylinders that are parallel at distance W, so S is the Shape factor of two long cylinders [4]. Making the temperature differences of both branches as following:

$$\theta_1 = T_1 - T_g$$

$$\theta_2 = T_2 - T_g$$

Therefore,

$$\frac{dT_1}{dx} = \frac{d\theta_1}{dx}$$

$$\frac{dT_2}{dx} = \frac{d\theta_2}{dx}$$

Substituting the above parameters into equations 1 and 2 and after some algebra, we obtain the following differential equations of temperature difference for each branch of U-tube:

$$\theta_1'(x) + [a+b]\theta_1(x) - b\theta_2(x) = 0 \ldots\ldots(3)$$

$$\theta_2'(x) - [a+b]\theta_2(x) + b\theta_1(x) = 0 \ldots\ldots(4)$$

Where

$$\theta' = \frac{d\theta}{dx}$$

$$a = \frac{h2\pi r_t}{\dot{m} c_p}$$

$$b = \frac{2\pi k_{gr}}{\dot{m}C_p \cosh^{-1}\left[\frac{W^2}{2r_t^2} - 1\right]}$$

Where x ranges from 0 to H. This ODE's system can be solved by finding out the eigenvalues and eigenvectors of the system. So the solution of these two equations is

$$\theta_1(x) = \beta\left[\left(\xi_2 - 1\right)e^{x\omega} + \left(1 - \xi_1\right)e^{(L-x)\omega}\right] \ldots\ldots(5)$$

$$\theta_2(x) = \beta\left[(\xi_2 - 1)\xi_1 e^{x\omega} + (1 - \xi_1)\xi_2 e^{(L-x)\omega}\right] \text{.....(6)}$$

Where

$$\beta = \frac{\theta_i}{(1-\xi_1)e^{L\omega} + (\xi_2 - 1)}$$

$$\theta_i = T_i - T_g$$

$$\omega = \sqrt{a(a+2b)}$$

$$\xi_1 = \frac{a+b+\omega}{b}$$

$$\xi_2 = \frac{a+b-\omega}{b}$$

$$L = 2H$$

2.2. Model

As an example, we will consider a U-tube with the following specifications shown in table 2

TABLE.2 CONDUCTIVITY OF THE MATERIALS AND THE RADIUS FOR EACH ZONE OF THE SYSTEM

Material	Heat Conductivity (W/m.K)	Zone Radius (m)
U-tube Pipe	0.33	0.0301
Grout	1.8	0.1
Soil	2.5	5

The radius of the U-tube is calculated by taking the average of inner and outer radius of the tube. The ground and fluid entrance temperatures are Tg = 293 K and Ti = 283 K. The following Table 3 shows the properties of the water which is taken at 10 C°.

TABLE .3 PROPERTIES OF WATER

Property	Value
C_p	4200 J/kg.K
P	1001.5 kg/m³
K	0.5745 W/m.K
μ	14x10exp-4
Pr	10.04

The following figure 4 shows the dimensions of the cross section of the model.

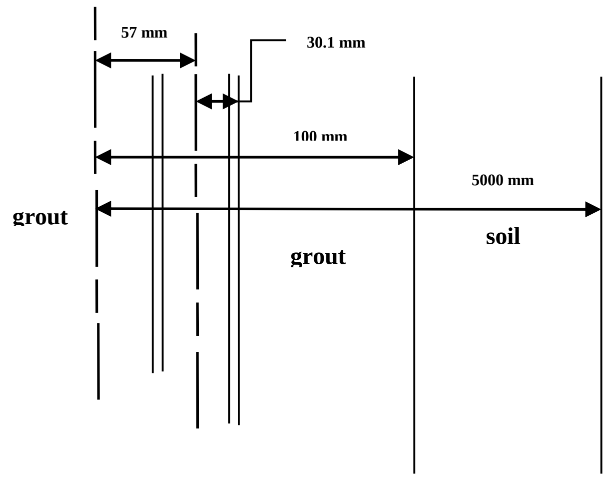

Fig. 4 Cross- Section of the Model

The distance between the two branches is 0.114 m. As consequence, the total heat transfer coefficient (h) can be obtained from the above information h = 14.88 W/m².

Fig.5 represents the analytical solution using equations (5&6). It shows how the temperature changes along a certain length (120m) and for different mass flow rates. As we can see, the out temperature of water decreases dramatically as we increase the flow rate. Consequently, operating the system at low flow rate, working fluid gains most of the surrounding ground heat. Using the analytical equations (5&6), we will get the same results that we have obtained from the numerical equation. Figure (13) indicates the change of the temperatures along a certain flow rate and length of the U-tube (0.2 kg/sec, 120 m). We can notice that it is the same result that we have in the numerical solution when the flow rate is 0.2 kg/sec. The result of the temperatures is in the appendix table.1.

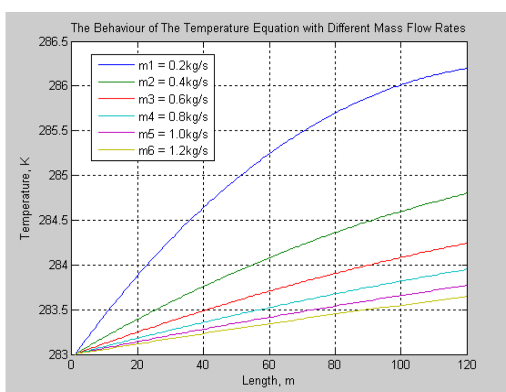

Fig. 5 The behavior of temperature equations (5&6) for different flow rates (120 m the length of the U-tube)

3. Entropy generation in the heat exchanger

The maximization of energy utilization and thus improvement in performance of thermal systems is one of the fundamental problems of engineering processes. One of the methods used for predicting the performance of an engineering process is the analysis of the system based on the second law of thermodynamics. The major reason for reduction in performance of a system is the existence of irreversibility. The entropy generation rate is a measurement of irreversibility. In any fluid flow system, irreversibility arises due to heat transfer and viscous dissipation of the fluid Bejan [5] in 1988. His analysis was based on that the passage of heat exchanger receives heat from one source. This passage is aduct of arbitrary cross-section area A_s and arbitrary wetted perimeter P. The heat transfer rate per unit length q´ is transferred to the stream m˙. For steady state, q´ crosses the temperature gap ΔT formed between the wall temperature and the bulk temperature of the stream T. The stream flows with friction in the x direction, thus, the pressure gradient is – dP/dx > 0. Taking as a thermodynamic system a passage of length dx, the first and second laws state:

$$q' = \frac{dQ}{dx}$$

$$\dot{m}dh = q'dx$$

$$\frac{d\dot{S}_{gen}}{dx} = \dot{m}\frac{ds}{dx} - \frac{q'}{T_g} \geq 0$$

$$T_g = T + \Delta T$$

Where $\dot{S}_{gen}$ is the entropy-generation rate. For our case, there are two sources of transferred heat which are heat from the ground and between the branches (legs of the U-tube heat exchanger). Each leg of the U-tube is treated alone as one of the branches losses heat to the other one. From fig. 6, the control volume is taken around U-tube, so the ground heat transfer $q'_{g,1}$& $q'_{g,2}$ and the heat between the branches q'_b crosses the wall temperature T_w which is assumed to be equal to T_g. Hence, the second thermodynamic laws of both branches are

$$\frac{d\dot{S}_{gen,1}}{dx} = \dot{m}\frac{ds_1}{dx} - \frac{q'_{g,1}}{T_g} - \frac{q'_b}{T_g}$$

$$\frac{d\dot{S}_{gen,2}}{dx} = \dot{m}\frac{ds_2}{dx} - \frac{q'_{g,2}}{T_g} + \frac{q'_b}{T_g}$$

Using the next definitions, we will end up with the following statements of entropy-generation rates of both branches:

$$dh = Tds + \upsilon dP$$

$$\upsilon = \frac{1}{\rho}$$

$$dh = C_p dT$$

$$f = \frac{\rho D_h}{2G^2}\left(-\frac{dP}{dx}\right)$$

$$G = \frac{\dot{m}}{A}$$

$$D_h = \frac{4A}{P}$$

$$\frac{d\dot{S}_{gen,1}}{dx} = -\left(q'_{g,1} + q'_b\right)\left[\frac{\theta_1}{\left(\frac{\theta_1}{T_g}+1\right)T_g^{\ 2}}\right] + \frac{\dot{m}}{\rho\left(\frac{\theta_1}{T_g}+1\right)T_g}\left(-\frac{dP}{dx}\right)$$

$$\frac{d\dot{S}_{gen,2}}{dx} = \left(q'_b - q'_{g,2}\right)\left[\frac{\theta_2}{\left(\frac{\theta_2}{T_g}+1\right)T_g^{\ 2}}\right] + \frac{\dot{m}}{\rho\left(\frac{\theta_2}{T_g}+1\right)T_g}\left(-\frac{dP}{dx}\right)$$

The temperature differences θ_1 & θ_2 are negligible compared with the ground temperature T_g. Consequently, the terms $(\theta_1/T_g + 1)$ & $(\theta_2/T_g + 1) \approx 1$. Substituting equations 1&2 into the above equations, we will end up with the following equations of entropy-generation rate,

$$\frac{d\dot{S}_{gen,1}}{dx} = \left(\frac{\phi+\delta}{T_g^{\ 2}}\right)\theta_1^{\ 2} - \left(\frac{\delta}{T_g^{\ 2}}\right)\theta_1\theta_2 + \frac{\dot{m}}{\rho T_g}\left(-\frac{dP}{dx}\right)$$

$$\frac{d\dot{S}_{gen,2}}{dx} = \left(\frac{\phi+\delta}{T_g^{\ 2}}\right)\theta_2^{\ 2} - \left(\frac{\delta}{T_g^{\ 2}}\right)\theta_1\theta_2 + \frac{\dot{m}}{\rho T_g}\left(-\frac{dP}{dx}\right)$$

First two terms in the both equations represent the entropy-generation rate that is arisen by heat transfer, whereas the last terms are generated by friction. Last terms of the two equations can be modified. From previous definition of *f* and $\dot{m} = \rho \pi r_t^2 U$, the entropy equations become

$$\frac{d\dot{S}_{gen,1}}{dx} = \left(\frac{\phi+\delta}{T_g^{\ 2}}\right)\theta_1^{\ 2} - \left(\frac{\delta}{T_g^{\ 2}}\right)\theta_1\theta_2 + \frac{f\dot{m}^3}{\rho^2\pi^2 r_t^{\ 5} T_g}$$

$$\frac{d\dot{S}_{gen,2}}{dx} = \left(\frac{\phi+\delta}{T_g^{\ 2}}\right)\theta_2^{\ 2} - \left(\frac{\delta}{T_g^{\ 2}}\right)\theta_1\theta_2 + \frac{f\dot{m}^3}{\rho^2\pi^2 r_t^{\ 5} T_g}$$

The following term represents the work that should be done for overcoming the friction of the pipe.

$$\dot{w}'_{pump} = \frac{f\dot{m}^3}{\rho^2\pi^2 r_t^{\ 5}}$$

Invoking reliable correlation for the friction (*f*) in both fully developed laminar and turbulent flow.

Thus, for laminar flow, the friction factor is

$$f = \frac{64}{R_e} \ldots\ldots\ldots R_e < 2300$$

While the formula of the friction factor for fully developed turbulent flow is taken from Haaland. It is an explicit formula.

$$f \approx \frac{0.308642}{\left[\log\left(\frac{6.9}{R_e D_t} + \left(\frac{\varepsilon / D_t}{3.7}\right)^{1.11}\right)\right]^2}$$

Where ε is the roughness of the U-tube. The material of the U-tube is usually high density polyethylene (HDPE). The roughness of Polyethylene PE - Corrugated with smooth inner walls ranges from 0.009 to 0.015. For our design, we will use 0.009. The Reynolds number is function of mass flow rate and the radius of the U-tube as the properties of the working fluid (water) is assumed to be constant. Hence, the friction work is responsible for making the flow either laminar or turbulent. As these equations show that the term entropy generations arisen by heat transfer are the same in both cases laminar and turbulent. The only difference is that term of entropy generation that is arisen by friction of the flow. So the entropy generation rate equations become:

$$\frac{d\dot{S}_{gen,1}}{dx} = \left(\frac{\phi + \delta}{T_g^{\ 2}}\right)\theta_1^{\ 2} - \left(\frac{\delta}{T_g^{\ 2}}\right)\theta_1\theta_2 + \frac{\dot{w}'_{pump}}{T_g}$$

$$\frac{d\dot{S}_{gen,2}}{dx} = \left(\frac{\phi + \delta}{T_g^{\ 2}}\right)\theta_2^{\ 2} - \left(\frac{\delta}{T_g^{\ 2}}\right)\theta_1\theta_2 + \frac{\dot{w}'_{pump}}{T_g}$$

With the purpose of obtaining the total entropy generation rate for the U-tube, we need to integrate the above equations with respect to (x). The total entropy generation equations have two terms, each term accounting for one irreversibility mechanism. These terms represents the entropy generation that are arisen by heat transfer in both branches, whereas the next term are the entropy generation that is due to friction.

$$\dot{S}_{gen,1,\Delta T} = \left(\frac{\beta^2(\phi+\delta)}{2\omega T_g^2}\right)[e^{2H\omega}(2\xi_1 - \xi_1^2 + \xi_2^2 - 2\xi_2 - 4H\omega(\xi_1 - 1)(\xi_2 - 1)\ldots. + e^{4H\omega}(\xi_1 - 1)^2] - \left(\frac{\beta^2\delta}{2\omega T_g^2}\right)[e^{2H\omega}(\xi_1(\xi_2^2 + 1 - \xi_1\xi_2)\ldots$$
$$-2H\omega(\xi_1 - 1)(\xi_2 - 1)(\xi_1 + \xi_2) - \xi_2)\ldots$$
$$+\xi_2 e^{4H\omega}(\xi_1 - 1)^2 - \xi_1(\xi_2 - 1)^2]\ldots(7)$$

$$\dot{S}_{gen,2,\Delta T} = \left(\frac{\beta^2(\phi+\delta)}{2\omega T_g^2}\right)[e^{2H\omega}(\xi_1(\xi_2^2 + 1 - \xi_1\xi_2) - 4H\omega(\xi_1 - 1)(\xi_2 - 1)\xi_1\xi_2\ldots. + \xi_1^2 - \xi_2^2 - 2\xi_1\xi_2^2 - 2\xi_2\xi_1^2) + (\xi_1 - 1)^2\xi_2^2 e^{4H\omega} - \xi_1^2(\xi_2 - 1)^2]$$

$$-\left(\frac{\beta^2\delta}{2\omega T_g^2}\right)[e^{2H\omega}(\xi_1(\xi_2^2+1-\xi_1\xi_2)-2H\omega(\xi_1-1)\ldots$$
$$(\xi_2-1)(\xi_1+\xi_2)-\xi_2)+(\xi_1-1)^2\xi_2 e^{4H\omega}\ldots$$
$$-\xi_1(\xi_2-1)^2]\ldots(8)$$

$$\dot{S}_{gen,\Delta T}=\frac{\dot{w}'_{pump}}{T_g}$$

Where

$$\delta=\frac{2\pi k_{gr}}{\cosh^{-1}\left[\frac{W^2}{2r_t^2}-1\right]}$$

$$\phi=h2\pi r_t$$

$\dot{S}_{gen,1,\Delta T}$ and $\dot{S}_{gen,2,\Delta T}$ are the entropy generation rates that are generated by heat transfer in branch (1) and (2) respectively. $\dot{S}_{gen,\Delta P}$ is the entropy generation rate that is generated by friction of the fluid flow. The fiction depends on the Reynolds number as the correlation formulas shows, so that makes the flow either laminar or turbulent. The total entropy generation rates for both branches are

$$\dot{S}_{gen,1}=\dot{S}_{gen,1,\Delta T}+\dot{S}_{gen,\Delta P}\ldots(9)$$

$$\dot{S}_{gen,2}=\dot{S}_{gen,2,\Delta T}+\dot{S}_{gen,\Delta P}\ldots(10)$$

For the whole U-tube heat exchanger

$$\dot{S}_{gen,UT}=\dot{S}_{gen,1}+\dot{S}_{gen,2}\ldots(11)$$

The above equations show that the total entropy generation rate in the U-tube heat exchanger. These equations rely on three parameters which are flow rate ($\dot{m}$), radius r_t and depth of borehole (H) or the length of the pipe. The parameters are related to the cost of the U-tube heat exchanger, so for the sake of setting economical system we need to find out the optimal parameters.

Considering the U-tube as a control volume and applying the first law of thermodynamic, the heat transfer rate per length for the U-tube can be written as following:

$$\dot{Q}_{H.Ex}=\dot{m}C_p(\theta_{out}-\theta_i)$$

By substituting the temperature distribution equations (5 & 6) into heat transfer equation of the U-tube, we obtain the following expression

$$\dot{Q}_{H.Ex} = \dot{m}C_p\beta(\xi_1 - 1)(\xi_2 - 1)(e^{\omega x} - e^{\omega(L-x)})$$

The total heat transfer from U-tube occurs where the inlet and outlet temperatures exchanges heat. So the total heat transfer from the U-tube is

$$\dot{Q}_{H.Ex} = \dot{m}C_p\beta(\xi_1 - 1)(\xi_2 - 1)(1 - e^{\omega L})$$

This equation relies on the length of the U-tube and the flow rate of working fluid. The heat transfer of the U-tube $q''_{H.Ex}$ is the heating load of the heat pump. From the equation of heat transfer, we can infer that there are variables that control the system which are flow rate ($\dot{m}$), the depth of borehole (H) or the length of the U-tube and heating load (Q_{max}). As a result, we can determine the optimized length of the U-tube from this equation if the desired heat transfer from the ground (Heating load of the pump) is available. Modifying the maximum heat transfer equation for the length of the U-tube, we obtain the next expression for the length of the u-tube or depth of borehole.

$$L = \frac{1}{\omega}\ln\left[\frac{\dot{m}C_p\theta_i(\xi_1 - 1)(\xi_2 - 1) - (\xi_2 - 1)Q_{\max}}{\dot{m}C_p\theta_i(\xi_1 - 1)(\xi_2 - 1) - (\xi_1 - 1)Q_{\max}}\right]$$

Where

$$L = 2H$$

Regarding to the above expression for the length, the length can be determined for different mass flow rates and heat loads, so it is a design condition. Next figures show how the length varies with these two input variables.

4. Results

The properties of the water was mentioned in table[2]. The ground and fluid entrance temperatures are Tg = 293 K and Ti = 283 K. Now, using equation (15, 16, 17 & 19) for different heating loads (1, 2, 3 & 4kw) and different flow rates at a constrained radius of the U-tube (0.00635, 0.0301, 0.0508 m), we obtain the following figures (6,7,8,9,10) and results:

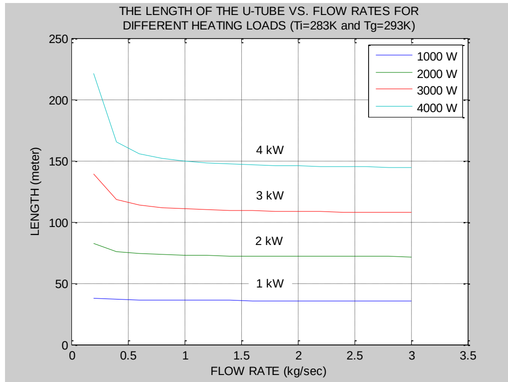

Fig. 6 The length of the U-tube vs. flow rate

The optimum U-tube length increases when the mass flow rate rises, whereas it decreases as the heating load does. Next figure (9) shows that it is better to use low mass flow rates since the increase in flow rates motivates the entropy generation to grow up. On the other hand, for the low heating loads, the entropy generation is less than that for the high heating loads.

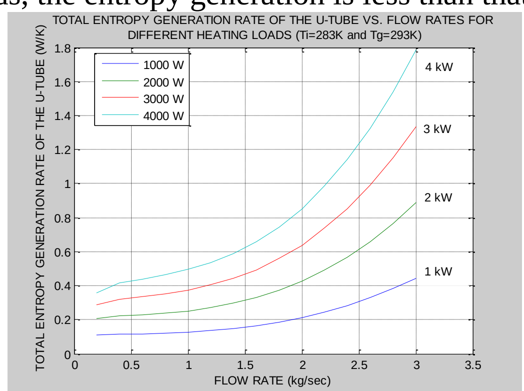

Fig. 7 The total entropy generation rates vs. flow rate

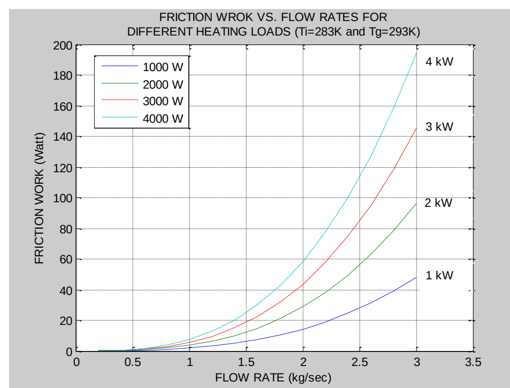

Fig.9 Friction work vs. flow rates

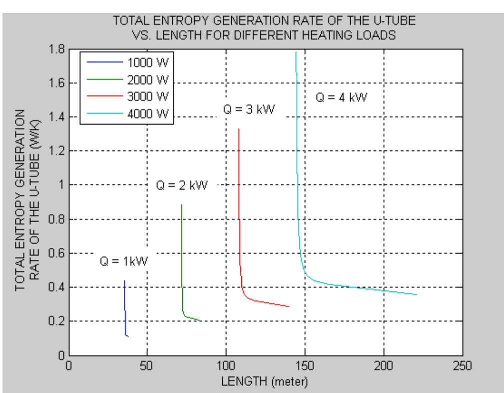

Fig.8 The total entropy generation rates vs. length

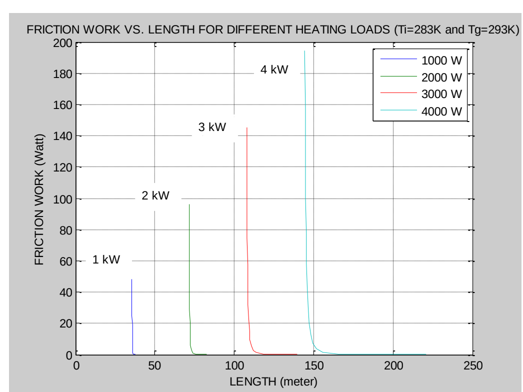

Fig.10 Friction work vs. lengths

From the above figures, we can conclude that increasing flow rate makes the entropy generation rate grow up in the U-tube. Also, the work that should be done for overcoming the friction increases with increasing the flow rate. As a result, using low flow rates ensure less entropy generation, therefore; less friction work. Meanwhile, for a constant heating load, the relationship between the optimum length and the flow rate is proportional. Low flow rate means that we need to use longer U-tube. For the assumed heating loads, we can obtain the optimum length of U-tube with radius (0.0301 m) for each load as shown in following Tables (4, 5, and 6).

TABLE.4 OPTIMIZED LENGTH FOR DIFFERENT HEATING LOADS
(RT = 0.0301M)

Optimum flow rate 0.2 kg/sec			
Heating Load (kW)	**Minimum Entropy Generation Rate (W/K)**	**Minimum Friction Work (W)**	**Optimum Length of The U-tube (m)**
1	0.1097	0.0158	37.9831
2	0.2055	0.0344	82.8067
3	0.2874	0.0580	139.5194
4	0.3556	0.0919	221.0436

To check the effects of U-tube radius to optimum length, we will use two other radius of U-tube (0.00635 & 0.0508 m).

TABLE.5 OPTIMIZED LENGTH FOR DIFFERENT HEATING LOADS
(RT = 0.00635M)

Optimum flow rate 0.2 kg/sec			
Heating Load (kW)	**Minimum Entropy Generation Rate (W/K)**	**Minimum Friction Work (W)**	**Optimum Length of The U-tube (m)**
1	1.0	200.0	52.6050
2	3.0	400.0	113.9411
3	5.0	700.0	189.0579
4	7.0	1000.0	288.7736

TABLE.6 OPTIMIZED LENGTH FOR DIFFERENT HEATING LOADS
(RT = 0.0508M)

Optimum flow rate 0.2 kg/sec			
Heating Load (kW)	**Minimum Entropy Generation Rate (W/K)**	**Minimum Friction Work (W)**	**Optimum Length of The U-tube (m)**
1	0.1096	0.0014	33.2127
2	0.2053	0.0032	73.9595
3	0.2871	0.0056	131.9633
4	0.3551	0.0111	260.1444

We can deduce from the above tables that using thinner radius of the U-tube increases the required work to overcome the friction and use longer optimum length. Finally, the bigger radius we use for U-tube, the less optimum length and required friction work. As a result, the relationship between the radius and length is inversely related. The optimal radius is 0.0508 meter for all heating loads.

5. Conclusion

Determination of optimum heat exchanger size is one of the most important parameters in the optimization of the heat exchanger design. In this project, optimum length for the heat exchanger is determined for different flow rates and heating loads by using the second law of thermodynamic. The optimal length minimizes entropy generation and therefore results in decrease the friction work and increase the efficiency of the heat pump. To sum up, for exploiting the heat ground effectively, a designer should be aware of the following notices:

- Using low flow rates reduce the entropy generation in the U-tube and required friction work, on the other hand, it will be at the expense of the length.
- The optimum length of the U-tube increases when the flow rates decreases.
- As a required maximum heating load increases, the length of the U-tube will be longer.
- Reduction of the heating load makes total entropy generation rate in the U-tube low
- When heating load is low, the required work for overcoming friction is low, too.
- Using thinner radius of the U-tube increases the optimum length and required friction work.

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