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Computational Method diffracted wave evolution at different sizes of obstacle's edge

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Abstract— **Electromagnetic propagating as movable energy may have various forms of manifestation depending on the source cycling speed rate by time. For normal centemetric and milimetric waves, wave shape dominates while for higher frequency usual form is light or rays, diffraction is a phenomena that occurs with most waves including the forms of electromagnetics. Analysis of such waves using 'Huygens' principle and well known forms of Fresnel approximations. Work in this paper is forwarded to computational code compute field strength resulting as diffracted wave at certain obstacle edge with different aperture size**

***Keywords*— Fresnel integrals, Computational Method, Diffraction, Incident.**

I. Introduction

Incident and reflection are usually main phenomena of wave trajectory when total mismatch occurs however by gradual improvement of properties of one or the two media to match with the other, part of the wave starts to sneak through the second media to form new type of wave called transmitted waves. However at certain situations where not all the second medium is facing the incident wave from first medium, but only an edge of it. This scenario will not allow chance for substational reflection nor and transmissions. Instead waves will be refracted around the edge.

This phenomenal is based on 'Huygens' principle which states that each point of a primary wave front can be considered to be the source of secondary spherical wave that propagates radially from that point. Fig.1 depicts the principle in physical optics while Fig.2 shows a plane wave incident upon a half plane (Conductor).

Diffraction is the phenomenon of a wave bending around an obstacle or through an aperture. It is a consequence of the wave nature of light, and is described by the Fresnel approximation. This approximation states that the diffracted wave can be expressed as a sum of two orthogonal functions, known as Fresnel integrals. These integrals are related to the angle of incidence and the size of the obstacle or aperture.

Fig.1 depicts the principle in physical optics

Fig.2 shows a plane wave incident upon a half plane (Conductor).

II. THEORETICAL FOUNDATIONS

A. The application of 'Huygens' principle states that the electric field at an arbitrary point of the secondary wave front envelope can be obtained from the following integral

$$E = \int_{\substack{over \\ x\to axis}} dE = \int_x^{\infty} E_o \frac{e^{-p(p+b)}}{p} dx \dots\dots\dots\dots\dots..(1)$$

Where dE is the differential electric field at point P due to the point source at a distance x from the origin O, as shown in Fig1. The term δ reflects the distance between two equiphase contours and it's expressed by:

$$\delta = \sqrt{p^2+x^2} - p = p\left[\left(1+\left(\frac{x}{2}\right)^2\right)^{\frac{1}{2}} - 1\right] \approx p\left[\left(1+\frac{x^2}{2p^2}\right) - 1\right] = \frac{x^2}{2p} \dots.(2)$$

Where we have assumed that δ≪p . Keeping the quadratic term in (2) is called the Fresnel approximation, while keeping the linear term only is called the Fraunhofer approximation. Geometric optics neglects the effects of diffraction and rays rather than the waves are considered. Defining q2 = 2/ρ⅄ and u = qx leads to the following expression for the electric field at the point of observation.

$$E = \frac{E_0}{p} e^{-jkp} \int_a^{\infty} e^{-jk8} dx = \frac{E_0}{p} e^{-jkp} \int_a^{\infty} e^{-j\frac{kx^2}{2p}} dx$$

$$= \frac{E_0}{qp} e^{-jkp} \int_{qa}^{\infty} e^{-i\frac{xu^2}{2}} du \dots\dots(3)$$

The last integral can be written as

$$E = \frac{E_0}{qp} e^{-jkp}\left[\int_0^{\infty} e^{-i\frac{xu^2}{2}} du - \int_0^{qa} e^{-j\frac{xu^2}{2}} du\right]..(4)$$

The integral in (4) are of the form of a well-studied integral in optics and are called Fresnel integrals. The solution of these integrals is given using the following expansion:

E=$\frac{E_0}{qp} e^{-i\mu p}\left\{\frac{1-j}{2} - \left|[C(qa) - jS(qa)]\right\}\right. \dots.5$

Where the Fresnel cosine integral and Fresnel sin integral are defined as:

$$C(x) = \int_0^x \cos\left(\frac{\pi 2t^2}{2}\right) dt..6$$

$$S(x) = \int_0^x \mathrm{SIN}\left(\frac{\pi 2t^2}{2}\right) dt..7$$

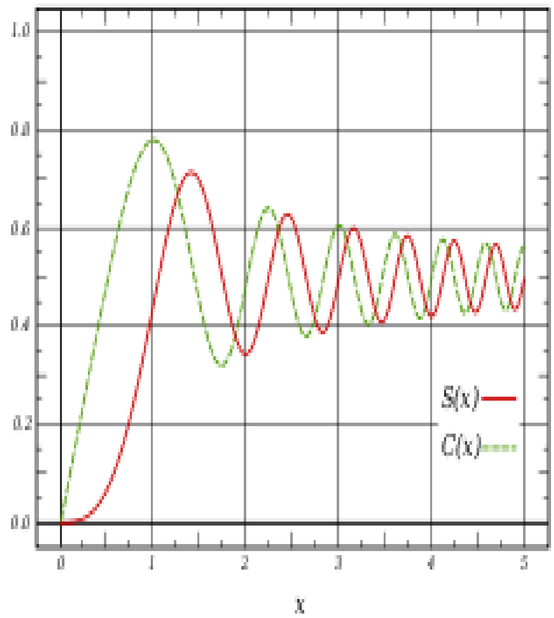

Fig3 shows Results Cosine &sine integrals

III. PTESTED EXAMPLES AND RESULT

All examples approximation is a mathematical technique used to describe the diffraction of electromagnetic waves around obstacles. It is based on the assumption that the wavefronts of the incident wave are planar and that the diffracted wavefronts are spherical. This approximation allows for an analytical solution to be obtained for the diffraction pattern..

A. Design example Fresnel integrals

A graph of S(qa) as a function of C(qa) as cornu's spiral and it is presented in fiure-4 parameter q= √ 2a/⅄ρ the MATLAB calculation of the Fresnel Integrals is based on the computation of the complex error function .Both the Fresnel cosine integral and the Fresnel sine integral are odd functions and their asymptotic values are ±0.5.

The average power density Sav in (W/m2) is calculated using the pointing vector we write.

$$Sav = \frac{|E^2|}{2Z^2} = S_0 \frac{1}{2}\left\{\left[\frac{1}{2} - C(qa)\right]^2 + \left|\left[\frac{1}{2} - S(qa)\right]^2\right\} \ldots \ldots 7$$

Cornu spiral showing the Fresnel integrals C(qa) and S(qa) as a function of the parameter Pa(-5<qa<5).The final asymptotic values as qa re C(qa)=S(qa)= ±0.5

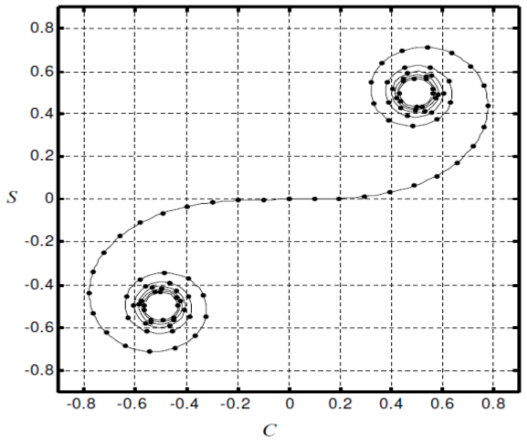

Fig4Cornu spiral showing Fresnel integrals (qa)&S(qa)

Where

$$S_0 = \frac{E_0{}^2 \lambda}{2Z_0 P}\frac{W}{m^2} \ldots \ldots \ldots 8$$

Is called the power density function of the incident wave at the point of observation if the obstacle were not present. The following coefficient is defined as afield diffraction coefficient D

$$D = \frac{E}{E_0} = \sqrt{\frac{1}{2}\left\{\left[\left[\frac{1}{2} - C(qa)\right]^2\right] + \left|\left[\frac{1}{2} - S(qa)\right]^2\right\}} \ldots .9$$

The MATLAB code has been designed and constructed to evaluate thase integrals for rang of q=-5 to +5 alsocan be used to plot these Fresnel Integrals.

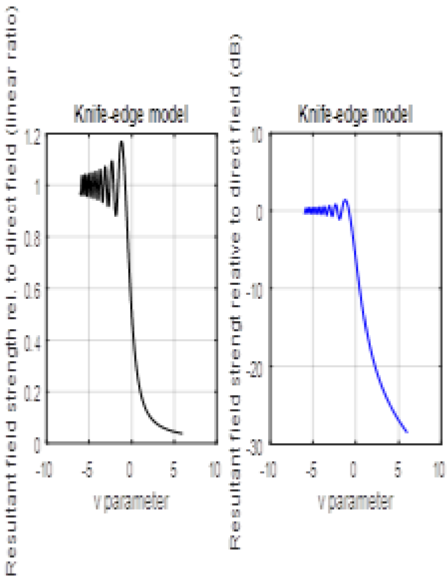

Fig5The KNIFE EDGE DIFFRACTION

B. Design example Circular Aperture Diffraction

When light from a point source passes through a small circular aperture, it produces a diffuse circular disc known as Airy's disc surrounded by much fainter concentric circular rings as an image as show in fig 6. This diffraction example is significant because the eye and many optical instruments have circular apertures. If the smearing of the image of the point source is greater than that caused by the system's aberrations, the imaging process is said to be diffraction-limited, and that is the best that can be done with that size aperture. This limitation on image resolution is quantified using the Rayleigh criterion, allowing the limiting resolution of a system to be calculated as show in fig7.

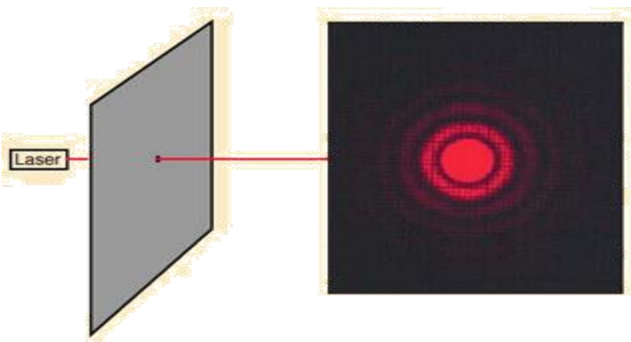

Fig6 show Circular Aperture Diffraction

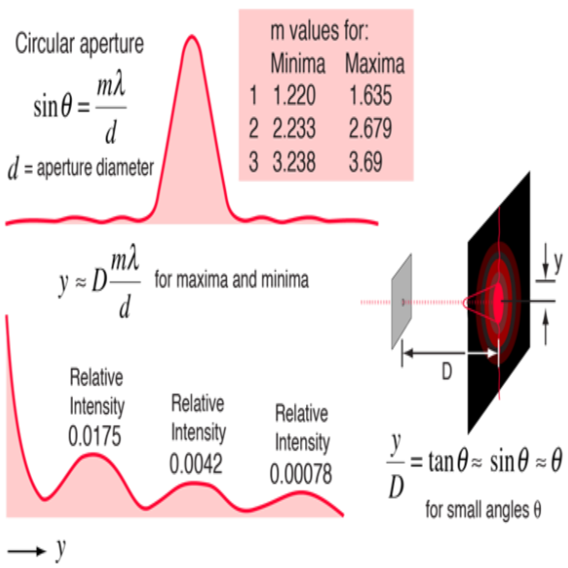

Fig7. The following pattern of maxima and minima in the diffracted intensity.

IV. CONCLUSIONS

Through the theoretical background and the design examples, it becomes obvious that refraction takes place through and any facing edge, through the refraction source geometry plays designated role.

Conclusions from this work leads to that fact that of waves or rays indenting on certain defined edges will continue through the refraction mechanism, However resulting refracted part needs consideration to evaluate behavior is values and shape.

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