Characterizations of Group Theory under Bipolar Neutrosophic Environment

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Abstract.

The study was conducted to introduce the concept of a bipolar neutrosophic subgroup and establish some of its basic properties and characterizations. Also present a similar application to the fundamentals of group theory. Which provides important background in this concept.

Keywords: Neutrosophic set; homomorphism; classical group; bipolar neutrosophic subgroup.

1- Introduction

To deal with uncertainty based on many complex systems like biological, behavioral and chemical etc. Zadeh [1] introduced the idea of concept fuzzy set as a constructive tool to handle uncertainties in many real applications. The traditional fuzzy set is characterized by the membership value or the grade of membership value. However, sometimes it may be difficult to assign the membership value for a fuzzy set. Consequently the concept of interval valued fuzzy set [2] was employed to solve the uncertainty of grade of membership value. In some real life problems in expert system, belief system, information fusion and so on, we must consider the truth-membership as well as the falsity-membership for proper description of an object in uncertain, ambiguous environment. Intuitionistic fuzzy set introduced by Atanassov [3] is appropriate for such a situation. neutrosophy concept was defined in 1999 by Smarandache [4] to deal with -indeterminate information and inconsistent information which exist commonly in real situations. In other words, "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra" [4]. In the neutrosophic set, a truth-membership, an indeterminacy-membership, and a falsity-membership are represented independently. The concept of neutrosophic set generalizes the above mentioned sets from philosophical point of view. From scientific and engineering point of view, the definition of neutrosophic set was specified by Wang et al. [5] which is called single valued neutrosophic set. The single valued neutrosophic set is a generalization of classical set, fuzzy sett and para-consistent set etc. Neutrosophic - set that is characterized - algebraic and topological directions (see [6-11]. Rosenfeld [12] studied the - concept of fuzzy subgroups in 1971 and then so many contributions were made on these main direction. Palaniappan et al. [13] gave the definition of intuitionistic L-fuzzy subgroup and studied some of its properties. In this study, we define a bipolar neutrosophic subgroup and give some of its properties. And study the so-called level subgroups of a bipolar neutrosophic subgroup. Moreover, we examine homomorphic image and preimage of a bipolar neutrosophic subgroup.

2- Preliminaries

In this section, we recall some definitions and basic results of neutrosophic set which will be used throughout the paper.

Definition1 [14]. A neutrosophic set A on U is $A =< T_A(x), I_A(x), F_A(x) >; x \in U$, where $T_A(x), I_A(x), F_A(x): A \to]-0, 1^+[$ and -0 < $T_A(x), I_A(x), F_A(x) < n3^+$.

Definition 2 [14]. Let N_1 and N_2 be two neutrosophic sets on E. Then,

$$1 - \ N_1 \cap N_2 =< \min\left(T_{N_1}(x), T_{N_2}(x)\right), \max\left(I_{N_1}(x), I_{N_2}(x)\right), \max\left(F_{N_1}(x), F_{N_2}(x)\right) >,$$

$$2 - \ N_1 \cup N_2 =< \max\left(T_{N_1}(x), T_{N_2}(x)\right), \min\left(I_{N_1}(x), I_{N_2}(x)\right), \min\left(F_{N_1}(x), F_{N_2}(x)\right) >.$$

Definition 3 [15]. A bipolar neutrosophic set A on U is $A =< {T_A}^+(x), {I_A}^+(x), {F_A}^+(x)$, ${T_A}^-(x), {I_A}^-(x), {F_A}^-(x) >; x \in U\}$ where ${T_A}^+(x), {I_A}^+(x), {F_A}^+(x): A \to [0, 1]$ and ${T_A}^-(x), {I_A}^-(x)$, ${F_A}^-(x): A \to [-1,0]$.

The positive membership degree ${T_A}^+(x), {I_A}^+(x), {F_A}^+(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree ${T_A}^-(x), {I_A}^-(x), {F_A}^-(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set *A*.

3- Main Result

In this section, we present a novel defnition of a bipolar neutrosophic group. In addition, we define the level set of the bibolar neutrosophic set.

Definition 4. Presume that G is a group. A bipolar neutrosophic subset is $A = \{< {T_A}^+(x), {I_A}^+(x), {F_A}^+(x), {T_A}^-(x), {I_A}^-(x), {F_A}^-(x) >; x \in G\}$, of G is called

a bipolar neutrosophic subgroup of G if the following axioms are satisfied:

(1) ${T_A}^+(xy) \geq \min({T_A}^+(x), {T_A}^+(y))$,

(2) ${T_A}^+(x^{-1}) \geq {T_A}^+(x)$

(3) ${I_A}^+(xy) \leq \max({I_A}^+(x), {I_A}^+(y))$,

(4) ${I_A}^+(x^{-1}) \leq {I_A}^+(x)$

(5) ${F_A}^+(xy) \leq \max({F_A}^+(x), {F_A}^+(y))$,

(6) ${F_A}^+(x^{-1}) \leq {F_A}^+(x)$

(7) ${T_A}^-(xy) \leq \max({T_A}^-(x), {T_A}^-(y))$,

(8) ${T_A}^-(x^{-1}) \leq {T_A}^-(x)$

(9) ${I_A}^-(xy) \geq \min({I_A}^-(x), {I_A}^-(y))$,

(10) ${I_A}^-(x^{-1}) \geq {I_A}^-(x)$

(11) ${F_A}^-(xy) \geq \min({F_A}^-(x), {F_A}^-(y))$,

(12) ${F_A}^-(x^{-1}) \geq {F_A}^-(x)$ where $x, y \in G$.

Proposition 1. The intersection of the finite set of bipolar neutrosophic subgroups is a bipolar neutrosophic subgroup.

Proof. We only verify (3) and (4) axioms in Definition 4, as the other axioms are well-known.

3) $[\cap {I_{A_i}}^+](xy) = \inf\left[{I_{A_i}}^+(x,y)\right]$,

$\leq \sup\left[\max({I_{A_i}}^+(x), {I_{A_i}}^+(y))\right]$,

$= \max\left[\sup({I_{A_i}}^+(x)), \sup({I_{A_i}}^+(y))\right]$,

$= \max\left[[\cap {I_{A_i}}^+](x), [\cap {I_{A_i}}^+](y)\right]$,

4) $[\cap {I_{A_i}}^+](x^{-1}) = \sup[{I_{A_i}}^+(x^{-1})]$,

$\leq \sup {I_{A_i}}^+(x)$,

$= [\cap {I_{A_i}}^+](x)$.

where *i*= 1, 2, . . . , *n*. Hence, the proposition is claimed

Proposition 2. Let A be a bipolar neutrosophic subgroup of G**.**

(1) ${T_A}^+(x^{-1}) = {T_A}^+(x)$ and ${T_A}^+(x) \leq {T_A}^+(e)$,

(2) ${I_A}^+(x^{-1}) = {I_A}^+(x)$ and ${I_A}^+(x) \geq {I_A}^+(e)$

(3) ${F_A}^+(x^{-1}) = {F_A}^+(x)$ and ${F_A}^+(x) \geq {F_A}^+(e)$,

(4) ${T_A}^-(x^{-1}) = {T_A}^-(x)$ and ${T_A}^-(x) \geq {T_A}^-(e)$,

(5) $I_A^-(x^{-1}) = I_A^-(x)$ and $I_A^-(x) \le I_A^-(e)$

(6) $F_A^-(x^{-1}) = F_A^-(x)$ and $F_A^-(x) \le F_A^-(e)$,

where $x \in G$ and e is an identity of G.

Proof. We only explain (4), as the other cases are widely known. Suppose that $x \in G$, then

$T_A^-(x) = T_A^-((x^{-1})^{-1}) \ge T_A^-(x^{-1}) \ge T_A^-(x)$

therefore $T_A^-(x) = T_A^-(x^{-1})$.

Also, we can prove$T_A^-(x) \ge T_A^-(e)$,

$I_A^-(e) = I_A^-(xx^{-1}) \ge min(I_A^-(x), I_A^-(x^{-1})) = I_A^-(x)$.

Proposition 3. Let A be a bipolar neutrosophic subgroup of G**.**

(1) $\mathrm{T_A}^+(\mathrm{yx}^{-1}) = \mathrm{T_A}^+(\mathrm{e})$ and $\mathrm{T_A}^+(\mathrm{y}) = \mathrm{T_A}^+(\mathrm{x})$,

(2) $\mathrm{I_A}^+(\mathrm{yx}^{-1}) = \mathrm{I_A}^+(\mathrm{e})$ and $\mathrm{I_A}^+(\mathrm{y}) = \mathrm{I_A}^+(\mathrm{x})$

(3) $\mathrm{F_A}^+(\mathrm{yx}^{-1}) = \mathrm{F_A}^+(\mathrm{e})$ and $\mathrm{F_A}^+(\mathrm{y}) = \mathrm{F_A}^+(\mathrm{x})$,

(4) $\mathrm{T_A}^-(\mathrm{yx}^{-1}) = \mathrm{T_A}^-(\mathrm{e})$ and $\mathrm{T_A}^-(\mathrm{y}) = \mathrm{T_A}^-(\mathrm{x})$,

(5) $\mathrm{I_A}^-(\mathrm{yx}^{-1}) = \mathrm{I_A}^-(\mathrm{e})$ and $\mathrm{I_A}^-(\mathrm{y}) = \mathrm{I_A}^-(\mathrm{x})$

(6) $\mathrm{F_A}^-(\mathrm{yx}^{-1}) = \mathrm{F_A}^-(\mathrm{e})$ and $\mathrm{F_A}^-(\mathrm{y}) = \mathrm{F_A}^-(\mathrm{x})$,

where $\mathrm{x}, \mathrm{y} \in \mathrm{G}$ and e is an identity of G.

Proof. We only explain (2), as the other cases are widely known. Presume that $x, y \in G$ and e is an identity of G. then

$I_A^+(y) = I_A^+\big((yx^{-1})x\big)$,

$\le max(I_A^+(yx^{-1}), I_A^+(x))$,

$= I_A^+(x)$.

On the other hand,

$I_A^+(x) = I_A^+\big((xy^{-1}), y\big)$,

$\le max(I_A^+(xy^{-1}), I_A^+(y))$,

$= I_A^+(y)$. Thus $I_A^+(y) = I_A^+(x)$.

Proposition 4. Any two bipolar neutrosophic subgroups that are union do not form a bipolar neutrosophic subgroup.

Proposition 5. Let G_p be a cyclic group with order p (p is a prime), then A is a bipolar neutrosophic subgroup of G_p if the following is held:

$T_A^+(x) = T_A^+(1) \geq T_A^+(0)$, $I_A^+(x) = I_A^+(1) \leq I_A^+(0)$ and $F_A^+(x) = F_A^+(1) \leq T_A^+(0)$ $T_A^-(x) = T_A^-(1) \leq T_A^-(0)$, $I_A^-(x) = I_A^-(1) \geq I_A^-(0)$ and $F_A^-(x) = F_A^-(1) \geq F_A^-(0)$ for all $x \in G_p$ and $x \neq 0$.

Proof. Assume the above conditions are satisfied, then all axioms in Definition 4 are verified, so

A is a bipolar neutrosophic subgroup of G_p. Conversely, for any $x \neq 0$and $y \neq 0$, inG_p it holds that $x = my$, and $y = nx$ where m, n are integers. Therefore, we have

$T_A^+(x) \geq T_A^+(y) \geq T_A^+(x)$, $I_A^+(x) \leq I_A^+(y) \leq I_A^+(x)$ and $F_A^+(x) \leq F_A^+(y) \leq T_A^+(x)$ $T_A^-(x) \leq T_A^-(y) \leq T_A^-(x)$, $I_A^-(x) \geq I_A^-(y) \geq I_A^-(x)$ and $F_A^-(x) \geq F_A^-(y) \geq F_A^-(x)$. The proposition is claimed.

Level Subgroups

We define the terminology level set of the bipolar neutrosophic subset.

Definition 5. Let A is a bipolar neutrosophic subset of B. For $\alpha \in [0,1], \alpha^- \in [-1,0]$,

$A_{\alpha,\alpha^-} = \{< T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x) >; x \in G: T_A^+(x) \geq \alpha, I_A^+(x) \leq \alpha,$

$F_A^+(x) \leq \alpha,\ T_A^-(x) \leq \alpha^-, I_A^-(x) \geq \alpha^-, F^-(x) \geq \alpha^-$ is labeled as a level subset of A.

Example 2. Consider the classical group$z_3 = \{0,1,2\}$ under addition modulo 3. We define a bipolar neutrosophic subgroup A of z_3 as follows:

$A = \{< 0,0.9, 0.1,0.3, -0.8, -0.2, -0.1 >, < 1,0.4, 0.6,0.7, -0.6, -0.8, -0.7 >,$

$< 2,0.4, 0.6,0.7, -0.8, -0.9, -0.8 >\}$.

Let $\alpha = 0.5$ and $\alpha^- = -0.5$, then by Definition5 we get a level subset of A as follows:

$A_{\alpha,\alpha^-} = \{< 0,0.9, 0.1,0.3, -0.8, -0.2, -0.1 >\}$. Since $T_A^+(0) = 0.9 \geq 0.5$, $I_A^+(x) = 0.1 \leq 0.5$, $F_A^+(0) = 0.3 \leq 0.5$, $T_A^-(0) = -0.8 \leq -0.5$, $I_A^-(0) = -0.2 \geq -0.5$, $F^-(0) = -0.1 \geq -0.5$. Then $A_{\alpha,\alpha^-}$is a subgroup of G.

Theorem 1. Presume that G is a group with identity e and A is a bipolar neutrosophic subgroup of G, then the level subset A_{α,α^-}, for $\alpha \in [0,1], \alpha^- \in [-1,0]$,

$T_A^{+}(e) \geq \alpha, I_A^{+}(e) \leq \alpha,\ \ F_A^{+}(e) \leq \alpha, , T_A^{-}(e) \leq \alpha^{-}, I_A^{-}(e) \geq \alpha^{-}, F^{-}(e) \geq \alpha^{-}$ is a subgroup of G.

Proof. Clearly, A_{α,α^-} is nonempty. Suppose that $x, y \in A_{\alpha,\alpha^-}$ then

$T_A^{+}(x) \geq \alpha, I_A^{+}(x) \leq \alpha,\ \ F_A^{+}(x) \leq \alpha, , T_A^{-}(x) \leq \alpha^{-}, I_A^{-}(x) \geq \alpha^{-}, F^{-}(x) \geq \alpha^{-}$

$T_A^{+}(y) \geq \alpha, I_A^{+}(y) \leq \alpha,\ \ F_A^{+}(y) \leq \alpha, T_A^{-}(y) \leq \alpha^{-}, I_A^{-}(y) \geq \alpha^{-}$and $F^{-}(y) \geq \alpha^{-}$.

Since A is a bipolar neutrosophic subgroup of G, then the axioms (1), (3), and (5) in Definition 4 are satisfied. This leads to

$T_A^{+}(xy) \geq \alpha, I_A^{+}(xy) \leq \alpha,\ \ F_A^{+}(xy) \leq \alpha,\ T_A^{-}(xy) \leq \alpha^{-}, I_A^{-}(xy) \geq \alpha^{-}$, and $F^{-}(xy) \geq \alpha^{-}$.

Hence $< xy, T_A^{+}(xy), I_A^{+}(xy), F_A^{+}(xy), T_A^{-}(xy), I_A^{-}(xy), F_A^{-}(xy) >; \in A_{\alpha,\alpha^-} >$. Also, since A neutrosophic subgroup of G, then the axioms (2), (4), and (8) in Definition 1.

are satisfied, and this leads to $T_A^{+}(x^{-1}) \geq \alpha, I_A^{+}(x^{-1}) \leq \alpha,\ \ F_A^{+}(x^{-1}) \leq \alpha, T_A^{-}(x^{-1}) \leq \alpha^{-}, I_A^{-}(x^{-1}) \geq \alpha^{-}$, and $F^{-}(x^{-1}) \geq \alpha^{-}$. This means

$< x^{-1}, T_A^{+}(x^{-1}), I_A^{+}(x^{-1}), F_A^{+}(x^{-1}), T_A^{-}(x^{-1}), I_A^{-}(x^{-1}), F_A^{-}(x^{-1}) >; \in A_{\alpha,\alpha^-} >$. Therefore, A_{α,α^-} is a subgroup of G.

Theorem 2. Presume that G is a group with identity e and A be a Bipolar neutrosophic subset of G, such that A_{α,α^-} is a subgroup of A for all $\alpha \in [0,1], \alpha^{-} \in [-1,0]$, , and $\alpha \leq T_A^{+}(e), \alpha \geq I_A^{+}(e),\ \ \alpha \geq F_A^{+}(e),\ T_A^{-}(e) \leq \alpha^{-}, I_A^{-}(e) \geq \alpha^{-}, F^{-}(e) \geq \alpha^{-}$, A is a bipolar neutrosophic subgroup of G.

Proof. Suppose that $a, b \in$ G with $A(a) = \alpha = (\alpha_1 \alpha_1^{-})$ and $A(b) = \alpha = (\alpha_2, \alpha_2^{-})$. Then,

$a \in A_{\alpha_1\alpha_1^-}$ and $b \in A_{\alpha_2\alpha_2^-}$,i.e., $T_A^{+}(a) \geq \alpha_1, I_A^{+}(a) \leq \alpha_1,\ \ F_A^{+}(a) \leq \alpha_1, T_A^{-}(a) \leq \alpha_1^{-}, I_A^{-}(a) \geq \alpha_1^{-}, F^{-}(a) \geq \alpha_1^{-}$ and $T_A^{+}(b) \geq \alpha_2, I_A^{+}(b) \leq \alpha_2,\ \ F_A^{+}(b) \leq \alpha_2, T_A^{-}(b) \leq \alpha_2^{-}, I_A^{-}(b) \geq \alpha_2^{-}, F^{-}(a) \geq \alpha_2^{-}$. Assume $\alpha_1 \leq \alpha_2$ and $\alpha_1^{-} \leq \alpha_2^{-}$. Then, follows, $A_{\alpha_2\alpha_2^-} \subseteq A_{\alpha_1\alpha_1^-}$. So $\in A_{\alpha_1\alpha_1^-}$. Thus $a, b \in A_{\alpha_1\alpha_1^-}$, and since $A_{\alpha_1\alpha_1^-}$ is a subgroup of G, by hypothesis,

$ab \in A_{\alpha_1\alpha_1^-}$. Thus $T_A^{+}(ab) \geq \alpha_1 = \min\left(T_A^{+}(a), T_A^{+}(b)\right), I_A^{+}(ab) \leq \alpha_1 = \max\left(I_A^{+}(a), I_A^{+}(b)\right), F_A^{+}(ab) \leq \alpha_1 = \max\left(F_A^{+}(a), F_A^{+}(b)\right), T_A^{-}(ab) \leq \alpha_1^{-}=\max\left(T_A^{-}(a), T_A^{-}(b)\right), I_A^{-}(ab) \geq \alpha_1^{-} = \min\left(I_A^{-}(a), I_A^{-}(b)\right)$, and $F_A^{-}(ab) \geq \alpha_1^{-} = \min\left(F_A^{-}(a), F_A^{-}(b)\right)$. Then, suppose that $a \in G$, and let $A(a) = \alpha$, then $a \in A_{\alpha_1\alpha_1^-}$. Since $A_{\alpha_1\alpha_1^-}$ is a subgroup, $a^{-1} \in A_{\alpha_2\alpha_2^-}$. Therefore, $T_A^{+}(a^{-1}) \geq \alpha, I_A^{+}(a^{-1}) \leq \alpha,\ \ F_A^{+}(a^{-1}) \leq \alpha, , T_A^{-}(a^{-1}) \leq \alpha^{-}, I_A^{-}(a^{-1}) \geq \alpha^{-}$, and $F^{-}(a^{-1}) \geq \alpha^{-}$, and hence $T_A^{+}(a^{-1}) \geq T_A^{+}(a), I_A^{+}(a^{-1}) \leq I_A^{+}(a),\ \ F_A^{+}(a^{-1}) \leq F_A^{+}(a), T_A^{-}(a^{-1}) \leq T_A^{-}(a)$,

${I_A}^-(a^{-1}) \geq {I_A}^-(a)$, and $F^-(a^{-1}) \geq {F_A}^-(a)$. Therefore A bipolar neutrosophic is subgroup of G.

Theorem 3. Let X_1, X_2 be the classical groups and $g: X_1 \rightarrow X_2$ be group homomorphism. If A is a bipolar neutrosophic subgroup of X_1, then the image of A, $g(A)$ is a bipolar neutrosophic subgroup of X_2.

Proof. Let A be a bipolar neutrosophic subgroup over X_1, and $y_1, y_2 \in X_2$, if $g^{-1}(A)(y_1) = \emptyset$, or $g^{-1}(A)(y_1)$, then it is obvious that $g(A)$ is a bipolar neutrosophic subgroup of X_2. Let us assume that there exist $x_1, x_2 \in X_1$, such that $g(x_1) = y_1$ and $g(x_2) = y_2$. Since g is a group homomorphism,

${T_A}^+(g(y_1, {y_2}^{-1})) = \vee_{y_1, {y_2}^{-1} = g(x)} {T_A}^+(x) \geq {T_A}^+(x_1, {x_2}^{-1})$,

${I_A}^+(g(y_1, {y_2}^{-1})) = \vee_{y_1, {y_2}^{-1} = g(x)} {I_A}^+(x) \geq {I_A}^+(x_1, {x_2}^{-1})$,

${F_A}^+(g(y_1, {y_2}^{-1})) = \wedge_{y_1, {y_2}^{-1} = g(x)} {F_A}^+(x) \leq {F_A}^+(x_1, {x_2}^{-1})$,

${T_A}^-(g(y_1, {y_2}^{-1})) = \wedge_{y_1, {y_2}^{-1} = g(x)} {T_A}^-(x) \leq {T_A}^-(x_1, {x_2}^{-1})$,

${I_A}^-(g(y_1, {y_2}^{-1})) = \wedge_{y_1, {y_2}^{-1} = g(x)} {I_A}^-(x) \leq {I_A}^-(x_1, {x_2}^{-1})$,

${F_A}^-(g(y_1, {y_2}^{-1})) = \vee_{y_1, {y_2}^{-1} = g(x)} {F_A}^-(x) \geq {F_A}^-(x_1, {x_2}^{-1})$,

By using the above inequalities let us prove that

$g(A)(y_1, {y_2}^{-1}) \geq g(A)(y_1) \wedge g(A)(y_2).\, g(A)(y_1, {y_2}^{-1})$,

$= (g({T_A}^+)(y_1, {y_2}^{-1}), g({I_A}^+)(y_1, {y_2}^{-1}), g({F_A}^+)(y_1, {y_2}^{-1}), g({T_A}^-)(y_1, {y_2}^{-1}), g({I_A}^-)$

$(y_1, {y_2}^{-1}), g({F_A}^-)(y_1, {y_2}^{-1}))$,

$= (\vee\, {T_A}^+(x), \vee\, {I_A}^+(x), \wedge\, {F_A}^+(x), \wedge\, {T_A}^-(x), \wedge\, {I_A}^-(x), \vee\, {F_A}^-(x))$,

$\geq {T_A}^+(x_1, {x_2}^{-1}), {I_A}^+(x_1, {x_2}^{-1}), {F_A}^+(x_1, {x_2}^{-1}), {T_A}^-(x_1, {x_2}^{-1}), {I_A}^-(x_1, {x_2}^{-1}), {F_A}^-(x_1, {x_2}^{-1}))$,

$\geq ({T_A}^+(x_1) \wedge {T_A}^+(x_2), {I_A}^+(x_1) \wedge {I_A}^+(x_2), {F_A}^+(x_1) \vee {F_A}^+(x_2), {T_A}^-(x_1) \vee {T_A}^-(x_2), {I_A}^-(x_1) \vee {I_A}^-(x_2), {F_A}^-(x_1) \wedge {F_A}^-(x_2)$,

$= ({T_A}^+(x_1), {I_A}^+(x_1), {F_A}^+(x_1), {T_A}^-(x_1), {I_A}^-(x_1), {F_A}^-(x_1)) \wedge ({T_A}^+(x_2), {I_A}^+(x_2), {F_A}^+(x_2),$

${T_A}^-(x_2), {I_A}^-(x_2), {F_A}^-(x_2))$,

This is satisfied for each $x_1, x_2 \in X_1$ with $g(x_1) = y_1$ and $g(x_2) = y_2$, then it is obvious that

$g(A)(y_1, {y_2}^{-1})$

$\geq (\vee_{y_1=g(x_1)} T_A{}^+(x_1), \vee_{y_1=g(x_1)} I_A{}^+(x_1), \wedge_{y_1=g(x_1)} F_A{}^+(x_1), \wedge_{y_1=g(x_1)} T_A{}^-(x_1), \wedge_{y_1=g(x_1)}$

$I_A{}^-(x_1), \quad \vee_{y_1=g(x_1)} F_A{}^-(x_1)) \wedge (\vee_{y_2=g(x_2)} T_A{}^+(x_2), \vee_{y_2=g(x_2)} I_A{}^+(x_2), \wedge_{y_2=g(x_2)} F_A{}^+(x_2),$

$\wedge_{y_2=g(x_2)} T_A{}^-(x_2), \wedge_{y_2=g(x_2)} I_A{}^-(x_2), \vee_{y_2=g(x_2)} F_A{}^-(x_2)),$

$= (g(T_A{}^+)(y_1), g(I_A{}^+)(y_1), g(F_A{}^+)(y_1), g(T_A{}^-)(y_1,), g(I_A{}^-)\,(y_1), g(F_A{}^-)(y_1)) \wedge$

$(g(T_A{}^+)(y_2), g(I_A{}^+)(y_2), g(F_A{}^+)(y_2), g(T_A{}^-)(y_2,), g(I_A{}^-)\,(y_2), g(F_A{}^-)(y_2))$

$= g(A)(y_1) \wedge g(A)(y_2).$

Hence the image of a bipolar neutrosophic subgroup is also a bipolar neutrosophic subgroup.

Theorem 4. Let X_1, X_2 be the classical groups and $g: X_1 \to X_2$ be group homomorphism. If B is bipolar neutrosophic subgroup of X_2, then the preimage $g^{-1}(B)$ is bipolar neutrosophic is subgroup of X_1.

Proof. Let B abipolar neutrosophic subgroup over X_2, and $x_1, x_2 \in X_1$, since g is group homomorphism, the following inequality is obtaind.

$g^{-1}(B)(x_1, x_2{}^{-1})$

$= T_B{}^+(g(x_1, x_2{}^{-1})), I_B{}^+(g(x_1, x_2{}^{-1})), \; F_A{}^+(g(x_1, x_2{}^{-1}), \; T_A{}^-(g(x_1, x_2{}^{-1}), I_A{}^-(g(x_1, x_2{}^{-1}),$

$F_A{}^-(g(x_1, x_2{}^{-1})).$

$\geq T_B{}^+(g(x_1)) \wedge T_B{}^+ g(x_2{}^{-1})), \; I_B{}^+(g(x_1)) \wedge I_B{}^+ g\,(x_2{}^{-1})), F_B{}^+(g(x_1)) \vee F_A{}^+(g\,(x_2{}^{-1})),$

$T_B{}^-(g(x_1)) \vee T_B{}^- g(x_2{}^{-1})), \; I_B{}^-(g(x_1)) \vee I_B{}^- g\,(x_2{}^{-1})), F_B{}^-(g(x_1)) \wedge F_A{}^-(g\,(x_2{}^{-1})),$

$= T_B{}^+(g(x_1), I_B{}^+(g(x_1)), \; F_A{}^+(g(x_1), \; T_A{}^-(g(x_1), I_A{}^-(g(x_1), F_A{}^-(g(x_1)) \wedge$

$T_B{}^+(g(x_2), I_B{}^+(g(x_2)), \; F_A{}^+(g(x_2), \; T_A{}^-(g(x_2), I_A{}^-(g(x_2), F_A{}^-(g(x_2)).$ Therefore

$g^{-1}(B)$ is bipolar neutrosophic subgroup of X_1.

4- Conclusions

The mathematical branches have recently found it useful and important to study neutrosophic sets. The definition of a bipolar neutrosophic group and its theory have been introduced by the author of this paper as an extension of a neutrosophic group. Also, we discussed normality of a bipolar neutrosophic subgroup of a classical group and studied its image and preimage under a group homomorphism.

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